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ORIGINAL ARTICLE

COMPETING RISK ANALYSIS OF LIFETIME DATA USING INVERSE MAXWELL DISTRIBUTION

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Abstract: The concept of competing risk arises in studies where failure of a system occurs due to one among several mutually exclusive causes. In this article, we consider the case when the lifetime of an individual or a component follows an inverse Maxwell distribution. In classical approach, we obtained the point, asymptotic confidence interval and boot-p interval estimation of the parameters of inverse Maxwell distribution. We also applied Bayesian approach under square error loss function to obtain point and highest posterior density interval estimation. For illustration, simulation result is established. Finally two real data sets are analyzed in support of study.

Key words: Bayesian Inference, Boot-p confidence Interval, Competing risk analysis, Highest posterior density interval.

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1. Introduction

Maxwell distribution has broad application in many fields such as in statistical physics, physical chemistry, and their related areas. Besides all these it has good number of applications in reliability theory also. The Maxwell distribution was first used as lifetime distribution by Tyagi and Bhattacharya (1989). Inferences based on generalized Maxwell distribution have been discussed by Chaturvedi and Rani (1998). Estimation of reliability characteristics for Maxwell distribution under Bayes paradigm was discussed by Bekker and Roux (2005). Radha and Vekatesan (2013) discussed the prior selection procedure in case of Maxwell probability distribution. Krishna and Malik (2012) obtained the Bayes estimators of parameters and reliability functions of Maxwell distribution under progressive type-II censoring scheme. Dey and Maiti (2010) proposed the Bayesian estimation of the parameter for the Maxwell distribution. Tomer and Panwar (2015) discussed the estimation procedure for the parameter of Maxwell distribution in the presence of progressive type-I hybrid censored data. Panwer *et al.* (2015) presented the competing risk data for Maxwell distribution for independent cause of failure. Later, Modi and Gill (2015) and Saghir and Khadim (2016) proposed lengths biased Maxwell distribution and discussed its various properties. Tomer (2016) discussed the maximum likelihood analysis of masked data under dependent competing risk generalized life distribution model and obtained the Maximum likelihood estimates of various parameters and masking probabilities using EM algorithm. Furthermore, several generalizations based on Maxwell distribution are advocated and statistically justified. Recently, two more extensions of Maxwell distribution have been introduced by Sharma *et al.* (2017, 2018) and discussed the classical as well as Bayesian estimation of the parameter along with the real-life application. Patel and Patel (2018) considered the double priors for the parameter of inverted exponential distribution and obtained the

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estimates of unknown parameter for quadratic loss function under Type-II censoring. Bhavsar and Patel (2019) discussed the Bayesian estimation for the parameters of the mixture of power function distribution under Type-II censoring. Kumar and Kumar (2020) discussed the classical and Bayesian estimation procedures of the parameters of inverse distribution by using randomly censored data. Tomer and Panwar (2020) had discussed inverse Maxwell distribution (InvMWD) with its statistical properties. They had obtained maximum likelihood estimates under classical and Bayesian approaches and discussed applicability of distribution in different fields.

A random variable X follows Maxwell distribution (MWD) if its probability density function is given by

$$f(x,\theta)=\frac{4}{\sqrt{\pi}}\frac{x^2}{\theta^{\frac{3}{2}}}e^{\frac{-x^2}{\theta}};\qquad x>0,\theta>0 \tag{1}$$

where, θ is the scale parameter.

Inverse maxwell distribution

If X has a Maxwell distribution then the random variable $Y=\frac{1}{X}$ is said to follow inverse Maxwell distribution (InvMWD). The pdf of inverse Maxwell distribution may be obtained by using the transformation. We have

$$f(y,\theta)=\frac{4}{\sqrt{\pi}}\frac{1}{y^4\theta^{\frac{3}{2}}}e^{\frac{-1}{\theta y^2}};\qquad y>0,\theta>0 \tag{2}$$

The survival function is given by

$$\bar{F}(t)=1-F(t)=\frac{2}{\sqrt{\pi}}\gamma\left(\frac{3}{2},\frac{1}{\theta t^2}\right), \tag{3}$$

where, $\gamma(a,z)=\int_0^z u^{a-1}e^{-u}\,du$ is lower incomplete gamma function. The hazard function, $h(y,\theta)=\frac{f(y,\theta)}{\bar{F}(y,\theta)}$, of InvMWD is upside down bathtub in nature *i.e.*, it increases sharply in initial phase then after reaching a peak point it deeps gradually and tending to zero. This means InvMWD represents the lifetime of such individuals/items which have an increasing chance of failing in early age of life span after survival up to a specific age, the rate of failure start decreasing as age increases.

2. Methodology

Suppose we have n such systems that each having k-components in series attachment. Here $Y_1,Y_2,\ldots,Y_n$ are the failure of all such systems where $Y_i=min(Y_{i1},Y_{i2}\ldots,Y_{ik});i=1,2,\ldots,n,k=1,2,\ldots,K$ and Y_{ik} represents the failure time of kth component of ith system. Then the competing risk is represented by $\{Y_i,S_i\};i=1,2,\ldots,n$, where S_i is the cause of failure of ith system. Then the likelihood is given by

$$L(\theta\mid\underline{y})=\prod_{k=1}^{K}\prod_{i=1;\in S_i^{(k)}}^{n_k}f(y_i,\theta_k)\prod_{i=1;l\neq k}^{K}\tilde{F}(y_i,\theta_i), \tag{4}$$

where $S_i^{(k)};k=1,2,\ldots,K$ indicates the system failed due to kth component and n_k denotes the number of components failed due to kth component. The likelihood comes out to be

$$L(\theta\mid\underline{y})=\prod_{k=1}^{K}\prod_{i=1;i\in S_i^{(k)}}^{n_k}\frac{4}{\sqrt{\pi}}\frac{1}{y_i^4\theta_k^{\frac{3}{2}}}exp\left(\frac{-1}{y_i^2\theta_k}\right)$$

$$\prod_{l=1;l\neq k}^{K}\frac{2}{\sqrt{\pi}}\gamma\left(\frac{3}{2},\frac{1}{\theta_l y_i^2}\right) \tag{5}$$

Here for the ease of the problem we are considering $K=3$. Then, taking logarithm of (5), we get the log likelihood as

$$l(\theta\mid\underline{d})\alpha-\frac{3n_1}{2}log(\theta_1)-\frac{3n_2}{2}log(\theta_2)-\frac{3n_3}{2}log(\theta_3)$$

$$-4\sum_{i=1}^{n}log(y_i)-\frac{1}{\theta_1}\sum_{i=1;i\in S_i^{(1)}}\frac{1}{y_i^2}-\frac{1}{\theta_2}\sum_{i=1;i\in S_i^{(2)}}\frac{1}{y_i^2}$$

$$-\frac{1}{\theta_3}\sum_{i=1;i\in S_i^{(3)}}^{n_3}\frac{1}{y_i^2}+\sum_{i=1,i\in S_i^{(3)}}^{n_1}log\gamma\left(\frac{3}{2},\frac{1}{\theta_2 y_i^2}\right)+\sum_{i=1;i\in S_i^{(1)}}^{n_1}log\gamma\left(\frac{3}{2},\frac{1}{\theta_3 y_i^2}\right)$$

$$+\sum_{i=1;i\in S_i^{(2)}}^{n_2}log\,\gamma\left(\frac{3}{2},\frac{1}{\theta_3 y_i^2}\right)+\sum_{i=1;i\in S_i^{(2)}}^{n_2}log\,\gamma\left(\frac{3}{2},\frac{1}{\theta_1 y_i^2}\right)$$

$$+\sum_{i=1;i\in S_i^{(3)}}^{n_3}log\,\gamma\left(\frac{3}{2},\frac{1}{\theta_1 y_i^2}\right)+\sum_{i=1\in S_i^{(3)}}^{n_3}log\,\gamma\left(\frac{3}{2},\frac{1}{\theta_2 y_i^2}\right) \tag{6}$$

Differentiating the log-likelihood partially with respect to $\theta_k;(k=1,2,3)$, and equating them to zero, we get the following expression which are used to obtain the MLEs of $\theta_k;(k=1,2,3)$, through numerical procedure.

$$\hat{\theta}_k = \frac{2}{3n_k}\sum_{i=1;i\in S_i^{(k)}}^{n_k}\frac{1}{y_i^2} - \frac{1}{\sqrt{\theta_k}}\sum_{l=1}^{K^*}\sum_{l\neq k}^{n_l}\frac{\frac{1}{y_l^3}e^{\frac{-1}{\theta_k y_l^2}}}{\gamma\left(\frac{3}{2},\frac{1}{\theta_k y_l^2}\right)} \quad (7)$$

where $K^* \in \{1,2,3\}$.

2.1 Asymptotic confidence intervals (ACIs)

The approximate (observed) asymptotic variance-covariance matrix for the MLE of parameters θ_1, θ_2 and θ_3 can be found by inverting $I(\underline{\hat{\theta}})$ as

$$I^{-1}(\lambda) = \begin{bmatrix} var(\hat{\theta}_1) & Cov(\hat{\theta}_1,\hat{\theta}_2) & Cov(\hat{\theta}_1,\hat{\theta}_3) \\ Cov(\hat{\theta}_2,\hat{\theta}_1) & var(\hat{\theta}_2) & Cov(\hat{\theta}_2,\hat{\theta}_3) \\ Cov(\hat{\theta}_3,\hat{\theta}_1) & Cov(\hat{\theta}_3,\hat{\theta}_2) & var(\hat{\theta}_3) \end{bmatrix}$$

Thus, using above equation we get the $100(1-\alpha)\%$ confidence limits for $\hat{\theta}_K$ $(k = 1, 2, 3)$ are given by

$$\hat{\theta}_k \mp Z_{\frac{\alpha}{2}}\sqrt{var(\hat{\theta}_k)},$$

where, $z_{\left(\frac{\alpha}{2}\right)}$ is upper $100\left(\frac{\alpha}{2}\right)$th percentile of standard normal variate. The required calculations are given below

$$\frac{\partial^2 I(\theta|y)}{\partial\theta^2} = \frac{3n_k}{2\theta_k^2} - \frac{2}{2\theta_k^3}\sum_{i=1}^{n_k}\frac{1}{y_i^2}$$

$$-\sum_{i=1}^{k*}\sum_{i\neq k}^{n_i}\psi\left(y_i,\hat{\theta}_k\right); K^* \in \{1,2,3\} \quad (8)$$

where,

$$\psi(y,\theta) = \xi(y,\theta)\left[\frac{1}{\theta^2 y^2} - \frac{5}{2\theta} + \xi(y,\theta)\right] \quad (9)$$

and

$$\xi(y,\theta) = \frac{1}{y^2}e^{\frac{-1}{\theta y^2}}\left[\theta^{\frac{5}{2}}\gamma\left(\frac{3}{2},\frac{1}{\theta y^2}\right)\right]^{-1} \quad (10)$$

All non-diagonal elements, $\frac{\partial^2 l(\underline{\theta}|\underline{d})}{\partial\theta_j\theta_k}$; $j\neq k,(j,k=1,2,3)$ are zero.

2.2 Boot-p Intervals

In case the number of sample observations in experiment is not very large ACIs mentioned above are not suitable. So, we provide another procedure to obtain bootstrap CIs for $\hat{\Theta}$ advocated by Efron and Tibshirani (1986). The steps for applying parametric bootstrap method are given below:

1. Based on the original sample $\underline{y} = (y_1, y_2, \ldots, y_n)$, obtain the MLE of $\hat{\Theta}$.
2. Under the same conditions generate sample, say $(x_1, x_2, \ldots, x_m)$, from the underlying distribution InvMWD $\{(\hat{\Theta})\}$.
3. Compute the MLE of $\hat{\Theta}$ based on $(x_1, x_2, \ldots, x_m)$, say $\hat{\Theta}^*$.
4. Repeat step (2) and (3) B times $^*_1, ^*_2, \ldots\ldots ^*_B$
5. Arrange $^*_1, ^*_2, \ldots\ldots ^*_B$ in ascending order.
6. A two-sided $100(1-\alpha)\%$ percentile bootstrap confidence interval of Θ, say $\left[^*_{L}, ^*_{U}\right]$ is given by $\left[^*_{L}, ^*_{U}\right]$ $\left[^*_{B\left(\frac{\alpha}{2}\right)}, ^*_{B\left(1-\frac{\alpha}{2}\right)}\right]$.

2.3 Bayesian estimation

Under the Bayesian paradigm θ is considered as a random variable. Let us consider inverted gamma distribution as prior distribution $IG(\mu_k, \nu_k)$ of θ_k where $k=1,2,3$, given by the pdf

$$\pi_k\theta_k \propto \frac{1}{\theta_k^{\nu_k+1}}exp\left\{-\frac{\mu_k}{\theta_k}\right\}; \quad \mu_k,\nu_k > 0, k=1,2,3. \quad (11)$$

Table 1: Average values of point estimate of θ_1 , θ_2 and θ_3 along their MSEs (in Bracket) for $n = 20$, 30 and n = 50.

	MLEs			Bayes Estimates		
n	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$
20	1.2110 (0.0990)	1.2323 (0.0912)	1.2852 (0.0992)	1.2813 (0.0300)	1.3032 (0.0243)	1.3295 (0.0220)
30	1.2220 (0.0740)	1.2640 (0.0718)	1.3076 (0.0786)	1.2778 (0.0289)	1.3078 (0.0251)	1.3323 (0.0230)
50	1.2210 (0.0480)	1.2750 (0.0568)	1.3233 (0.0523)	1.2660 (0.0248)	1.3019 (0.0245)	1.3381 (0.0211)

Table 2: Average length of interval estimates along with their coverage probabilities of θ_1 , θ_2 and θ_3 for $n = 20$, 30 and 50.

	ACI			Boot-t			HPD		
n	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$
20	1.4640 93.86	1.4189 86.99	1.6654 95.98	1.3516 97.12	1.3156 91.12	1.4521 95.45	0.8942 99.33	0.9007 99.67	0.9110 99.42
30	1.1081 93.13	1.084 89.12	1.2382 93.91	0.9008 95.13	0.9849 87.12	0.9381 96.9	0.8193 98.83	0.8308 98.58	0.8410 98.67
50	0.8224 93.75	0.8105 87.71	0.9085 92.58	0.7524 91.75	0.7405 89.71	0.8185 93.58	0.7107 98.5	0.7275 97.67	0.7448 96.25

Merging the joint prior density with the likelihood function, the required joint posterior, up to proportionality, comes out to be

$$\pi\left(\underline{\Theta} \mid \underline{y}\right) \propto \frac{1}{\theta_1^{\frac{3n_1}{2}+v_1+1}} \frac{1}{\theta_2^{\frac{3n_2}{2}+v_2+1}} \frac{1}{\theta_3^{\frac{3n_3}{2}+v_3+1}}$$

$$\prod_{i=1;i\in S_i^{(1)}}^{n_1} \frac{1}{y_i^4} e^{-\frac{1}{\theta_1}\left(\mu_1+\frac{1}{y_i^2}\right)} \gamma\left(\frac{3}{2},\frac{1}{\theta_2 y_i^2}\right)\gamma\left(\frac{3}{2},\frac{1}{\theta_3 y_i^2}\right)$$

$$\prod_{i=1;i\in S_i^{(2)}}^{n_2} \frac{1}{y_i^4} e^{-\frac{1}{\theta_2}\left(\mu_2+\frac{1}{y_i^2}\right)} \gamma\left(\frac{3}{2},\frac{1}{\theta_3 y_i^2}\right)\gamma\left(\frac{3}{2},\frac{1}{\theta_1 y_i^2}\right)$$

$$\prod_{i=1;i\in S_i^{(3)}}^{n_3} \frac{1}{y_i^4} e^{-\frac{1}{\theta_3}\left(\mu_3+\frac{1}{y_i^2}\right)} \gamma\left(\frac{3}{2},\frac{1}{\theta_1 y_i^2}\right)\gamma\left(\frac{3}{2},\frac{1}{\theta_2 y_i^2}\right) \quad (12)$$

From the above expression we observe that the marginal distributions of $\theta_k\,(k=1,2,3)$ cannot be obtained in the closed form, which is essential in order to obtain the Bayes estimates of individual parameters or to obtain the parametric functions. Therefore, for further analysis, we proceed to Gibbs Sampler. For this we have to obtain the full condition distributions for $\theta_k\,(k=1,2,3)$, which are given below as:

$$\pi_k\left(\theta_k \mid \underline{d}\right) \propto \frac{1}{\theta_k^{\frac{3n_k}{2}+v_k+1}} exp\left\{-\frac{1}{\theta_k}\left(\mu_k + \sum_{i=1;i\in S_i^{(k)}}^{n_k} \frac{1}{y_i^2}\right)\right\}$$

$$\prod_{l=1;l\neq k}^{n_k} \gamma\left(\frac{3}{2},\frac{1}{\theta_k y_l^2}\right) \quad (13)$$

2.4 MCMC Method

We use Metropolis-Hastings algorithm method with proposal density normal distribution to generate sample observations from $\pi_k\left(\theta_k \mid \underline{d}\right),(k=1,2,3)$ given in Equation (13). The Metropolis-Hastings Algorithm has the following steps:

1. Set $t=1$ and take $\theta_1^0 = \hat{\theta}_1$,$\theta_1^0 = \hat{\theta}_2$ and $\theta_3^0 = \hat{\theta}_3$.
2. Generate a candidate point θ_k^* from proposal density $q_k \sim N\left(\hat{\theta}_k, V\left(\hat{\theta}_k\right)\right)$ and take a point u from a uniform distribution U (0, 1).
3. Then compute an acceptance ratio

Table 3: Average MLE and bayes estimates of the parameters for θ_1, θ_2 and θ_3 for Hoel data.

	ML Estimates			Bayes Estimates		
	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$
Hoel Data	3.6311	1.6336	3.0978	3.6648	1.6521	3.1297
Abortion Data	22.9683	4.4925	25.5069	22.9325	4.5344	25.5673

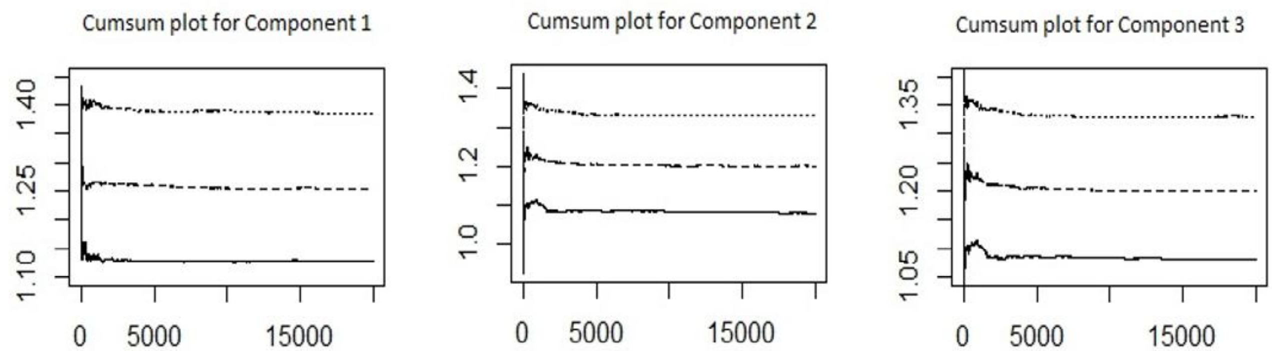

Fig. 1: Cumsum plot for quantiles at 5%, 50% and 95% for θ_1, θ_2 and θ_3

$$r_k = \frac{\pi_1\left(\theta_k^* \mid \underline{d}\right) q_k\left(\theta_k^{(t-1)}\right)}{\pi_1\left(\theta_k^{(t-1)} \mid \underline{d}\right) q_k\left(\theta_k^*\right)}$$

4. Let $p\left(\theta_k^{(t-1)}, \theta_k^*\right) = min\left(r_k, 1\right)$, then set $\theta_k^{(t)} = \theta_k^*$ if $u \le p\left(\theta_k^{(t-1)}, \theta_k^*\right)$ and otherwise $\theta_k^{(t)} = \theta_k^{(t-1)}$.
5. Set $t = t + 1$.
6. Repeat steps (2)-(4), N times to get the chain $\theta_1^1, \theta_1^2, \ldots, \theta_1^N$, $\theta_2^1, \theta_2^2, \ldots, \theta_2^N$ and $\theta_3^1, \theta_3^2, \ldots, \theta_3^N$, where N is very large number.

After the convergence of chain, we obtain N^* out of N samples, say $\theta_1^1, \theta_1^2, \ldots, \theta_1^{N^*}$, $\theta_2^1, \theta_2^2, \ldots, \theta_2^{N^*}$ and $\theta_3^1, \theta_3^2, \ldots, \theta_3^{N^*}$, with the help of these sample observations, we obtain the Bayes estimates for the parameters. We also obtain Bayesian credible and HPD intervals for θ by using algorithm given by Chen and Shao (1999).

3. Simulation Study

In this section, we perform the simulation study to verify theoretical results numerically and check the performance of estimators. Since we have considered the three-component series systems, so actual values taken in computation are $\theta_1 = 1.2, \theta_2 = 1.25$ and $\theta_3 = 1.3$. The simulation study is carried out for sample size $n = 20, 30$ and 50. In this section, we perform the simulation study to verify the theoretical results numerically and check the performance of estimators. Based on this, the MLE, ACI and Boot-p CIs are calculated. We repeat each generation and estimation procedures 1000 times and give average values of point estimates and corresponding mean square errors (MSEs) in Table 1. We also provide average length and coverage probabilities (CP) given in Table 2.

The Bayes estimates and HPD intervals are obtained using Gibbs Sampler in which samples are drawn from full conditional using Metropolis-Hasting algorithm. We run the three different chains by generating 50000 observations. For diagnosis of the convergence of chains, we draw Cumuplot at 5%, 50% and 95% quantiles. Outcomes of Cumuplot function applied to MCMC samples are given in Fig. 1 for θ_1, θ_2 and θ_3, respectively. For the arbitrarily chosen values of prior parameters $\mu_1 = 5, \mu_1 = 5, \mu_1 = 5, n_1 = 2, n_2 = 2$ and $n_3 = 2$, we present the average values of Bayes estimates along with their MSE's for 1000 repeated samples. The Bayes estimates and their MSEs are given in Table 1 and HPD with coverage probabilities are presented in Table 2.

Hoel Data: Here in our Analysis, we have used Hoel

Table 4: Upper and lower limit of interval estimates along with average length for hoel data.

	ACI			Boot-t			HPD		
	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$
Lower	3.3623	1.4893	2.8598	2.9879	1.4147	2.3243	3.1032	1.3299	2.5730
Upper	3.8999	1.7780	3.3357	4.3156	1.9060	3.9879	4.3935	1.9583	3.6737
Length	1.261	0.6767	1.1173	1.3114	0.4964	1.6881	1.2528	0.6622	1.0862

Table 5: Upper and lower limit of interval estimates along with average length for abortion data.

	ACI			Boot-t			HPD		
	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$
Lower	19.9282	3.7561	22.1785	20.0071	4.3376	21.3755	19.9561	3.9509	21.7069
Upper	26.0084	5.2288	28.8352	25.8676	4.6663	29.3898	25.7150	5.2218	28.1149
Length	6.0802	1.4727	6.6568	5.8605	8.0143	6.5258	5.7589	1.2709	6.4079

(1972) data. Hole's data, arise from a laboratory experiment in which male mice of strain RFM were given radiation dose 300 rats at 5 to 6 weeks old. There are two groups of mice: conventional lab environment (group 1) and germ-free environment (group 2). The survival times are measured in days and the causes of death are (1) (thymic lymphoma), (2) (reticulum cell sarcoma), and (3) (other). For calculation purpose data is scaled by dividing 1000. The point and interval estimates for Hoel data is given in Table 3 and 4.

Abortion data: Abortion data set is available in *R* package 'mvna'. The data is about the outcomes of pregnancies exposed to coumarin derivatives. The aim is to investigate whether exposition to coumarin derivatives increases the probability of spontaneous abortions. Apart from spontaneous abortion, pregnancy may end in induced abortion or live birth. The data contains 1186 observations of the 5 variables in two groups (control and exposed to coumarin derivatives) and the cause of failure are induced abortion, life birth and spontaneous abortion. We consider group 2 for the analysis purpose which contains the 173 observations. Exit times are taken as the time of events. For calculation purpose data is scaled by dividing 100. The estimates are given for the considered abortion data in Table 3 and 5.

4. Discussion and Conclusion

From the simulation study, it can be easily seen that as sample size increases the MSEs of the respective estimates decrease rapidly. As sample size increases average length of intervals also decreases with increasing coverage probability which is obvious and certify all theoretical derivation. ACI's are always symmetric, but boot-p and HPD CIs shows the actual nature of the parameter estimator. It can also be observed that the HPDs have shorter lengths then that of ACIs and Boot-t such that it provides us results in more precise manner. And we can recommend Bayesian inferential in place of classical estimation procedure.

Since the distribution with upside down bathtub hazard rate model are rarely used for competing risk analysis. So, all such experiments where individuals/ items in competing cause and follows upside down bathtub hazard rate, this study can be utilized easily. We can guaranty more accurate results rather than using any other model can achieve which is not appropriate for the system.

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