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ORIGINAL ARTICLE

AVAILABILITY ANALYSIS OF A THREE UNIT COLD STANDBY SYSTEM USING LINDLEY FAILURE AND REPAIR LAWS

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Abstract : Here, a stochastic model for a cold standby system of three identical units is developed by using single repair facility. The repair facility is called a server who attends the system immediately to rectify the faults which occur during system operation. The most important monotonic decreasing life time Lindley distribution is taken up for failure and repair time of the system with different parameters. The use of semi-Markov process and regenerative point technique is done to derive the expressions for mean time to system failure and availability of the system in steady state. The behaviour of the reliability measures has been observed graphically for arbitrary values of the shape parameter associated with the failure and repair times.

Key words : Three Unit System, Immediate Visit of the Server, Reliability Characteristics, Lindley Distribution.

1. Introduction

Several life time distributions including Exponential, Weibull, Gamma and Log-Normal distributions have been considered for failure and repair time of the repairable systems. As a result of which large amount of literature has been made available on stochastic modelling of repairable systems by considering these distributions particularly the negative exponential distribution. The generalizations of existing distribution is another interest in reliability engineering research and thus new models have been developed by extending existing distributions. The Lindley distribution has been suggested as a better fit than the exponential distribution for some life time data. And, Lindley distribution has several real applications where failure data show the non-monotone shape. Ghitany *et al*. (2008) discussed Lindley distribution and its applications. Over the years, the concentration of the reliability engineers is on the development of the faultless systems. But, failures are ways of life and cannot be ignored completely. Therefore, the purpose is not only to develop a reliability system but also to identify the reliability improvement techniques. The standby redundancy has been considered as one of the ways for improving system reliability. And, cold standby redundancy is considered as most effective technique to enhance the performance and durability of repairable systems. A lot of research papers have been reported on the reliability modelling of standby systems with different repair policies. The reliability measures of cold standby systems have been analyzed by Murari and Goyal (1984). Cao and Wu (1989) developed a two unit cold standby system with replaceable repair facility. Goel and Shrivastava (1991) discussed a two unit cold standby system with three modes and correlated failures and repairs. Malik and Nandal (2008) analyzed a system of three reliability models with the provision of spare unit. Chander and Singh (2009) developed a reliability model of 2-out-of-3 redundant system subject to degradation after repair. Kadyan *et al*. (2010) analyzed a redundant system subject to degradation and priority for operation to new unit.

In most of these studies the failure rate of the unit has been taken constant and follows negative exponential distribution. The system model has been analyzed by semi-Markov process and regenerative point technique. Also, the focus of the researchers was on the development of system models with two identical

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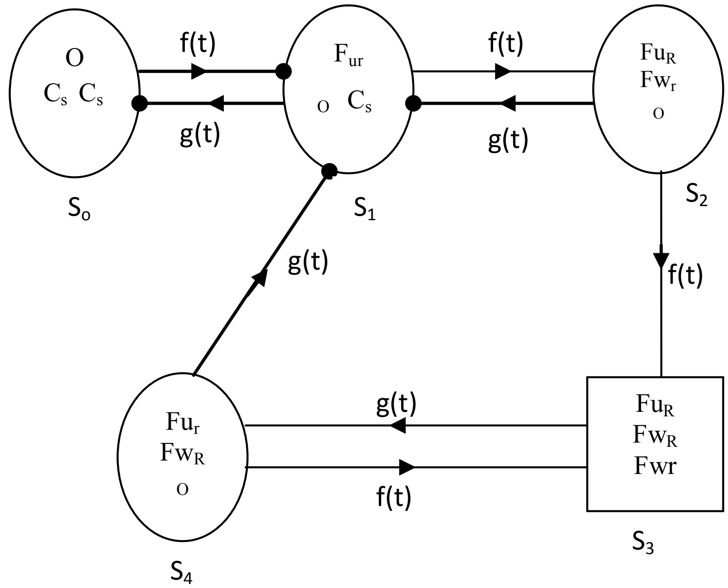

Fig. 1 : State Transition Diagram.

Where, ● Regenerative Point ◯ Operative Unit ☐ Failed State

units. There are many repairable systems where three or more units are required not only to increase the durability but also to cover the risk in emergency. For example, wireless sensor systems, where a sensor typically needs several spares because of harsh environments and high reliability requirement. The sensor may have limited processing capacity and memory, so it has to back up the acquired data to a server.

In view of the above facts and observations, the purpose of the present study is to analyze availability of a cold standby system of three identical units by considering Lindley distribution for failure and repair times. There are two modes of the unit- operative and complete failure. The repair activities are handled by a single server, who attends the system immediately whenever required. The repairs are perfect. The use of semi-Markov process and regenerative point technique is done to derive the expressions for mean time to system failure and availability of the system in steady state. The behaviour of the reliability measures has been observed graphically for arbitrary values of the shape parameter associated with the failure and repair times.

2. Lindley Distribution

A Lindley distribution is a way to describe the lifetime of a process or device. It is widely used in biology, engineering and medicine. Ghitany *et al*. (2008) discussed the Lindley distribution and its applications. The probability density function of Lindley distribution is given by

$$f(t)=\frac{\delta^2}{(\delta+1)}(1+t)e^{-\delta t},\ t>0,\ \delta>0$$

$$F(t)=1-\frac{(1+\delta+t)e^{-\delta t}}{(\delta+1)},\ t>0,\ \delta>0$$

The reliability function of Lindley distribution is given by

$$R(t)=P[X>t]=\frac{(1+\delta+t)e^{-\delta t}}{(\delta+1)},\ t>0,\ \delta>0$$

The hazard function of Lindley distribution is given by

$$h(t)=\frac{f(x)}{R(t)}=\frac{\delta^2(1+t)}{(1+\delta+t)}$$

3. System Description

Here, we discuss a reliability model for a system of three identical units with immediate visit of the server. The block diagram of the system model is shown in Fig. 1.

4. Notations

O : The unit is operative and in normal mode

Fu_r/Fu_R : The unit is failed and under repair/ Continuous from previous state

Fw_r/Fw_R : The unit is failed and waiting for repair/ Continued from previous state

S_o : The initial state in which the system is good and operative

S_1 : The second state in which one unit is under repair and system is operative

S_2 : The third state in which one unit is under repair, one unit is waiting for repair and one unit is operative

S_3 : The fourth state in which system is failed and waiting for repair

S_4 : The fifth state in which one unit is operative, one is under repair and one is waiting for repair

f(t)/F(t) : pdf/cdf of failure time of the operative unit

g(t)/G(t) : pdf/cdf of repair time of the failed unit

® : Laplace Stieltjes Convolution

5. Transition Probabilities

Simple probabilistic considerations yield the following expression for the non-zero elements

$p_{ij} = \lim_{t\to\infty} Q_{ij}(t) = \int_0^\infty q_{ij}(t)dt$ as

$$dQ_{01}(t)=\int_0^\infty q_{01}(t)dt=\int_0^\infty \frac{\theta^2}{(\theta+1)}(1+t)\,e^{-\theta t}dt \quad (1)$$

$$dQ_{10}(t)=\int_0^\infty q_{10}(t)dt=\int_0^\infty f(t)\overline{F(t)}dt=dQ_{21}(t)$$

$$= dQ_{41}(t) \quad (2)$$

$$dQ_{12}(t)=\int_0^\infty q_{12}(t)dt=\int_0^\infty f(t)\overline{G(t)}dt=dQ_{23}(t)$$

$$= dQ_{43}(t) \quad (3)$$

$$dQ_{34}(t)=\int_0^\infty q_{34}(t)dt=\int_0^\infty g(t)dt \quad (4)$$

where,

$$f(t)=\frac{\theta^2}{(\theta+1)}(1+t)e^{-\theta t} \qquad g(t)=\frac{\lambda^2}{(\lambda+1)}(1+t)e^{-\lambda t}$$

$$\overline{F(t)}=\frac{(1+\theta+t)e^{-\theta t}}{(\theta+1)} \qquad \overline{G(t)}=\frac{(1+\lambda+t)e^{-\lambda t}}{(\lambda+1)}$$

Taking Laplace Stieltjes Transform, we have

$$Q_{01}^{**}(s)=\int_0^\infty e^{-st}d[Q_{01}(t)]dt$$

$$=\int_0^\infty e^{-st}\left(\frac{\theta^2}{(\theta+1)}(1+t)e^{-\theta t}\right)=\frac{\theta^2(\theta+s+1)}{\left[(\theta+1)(\theta+s)^2\right]} \quad (5)$$

$$Q_{10}^{**}(s)=\frac{\lambda^2\left\{(\lambda+\theta+s)^2(\theta+1)+(\lambda+\theta+s)(1+2\theta)2\theta\right\}}{\left\{(\lambda+1)(\theta+1)+(\lambda+\theta+s)^3\right\}}$$

$$=Q_{21}^{**}(s)=Q_{41}^{**}(s) \quad (6)$$

$$Q_{12}^{**}(s)=\frac{\theta^2\left\{(\lambda+\theta+s)^2(\lambda+1)+(\lambda+\theta+s)(1+2\lambda)2\lambda\right\}}{\left\{(\lambda+1)(\theta+1)+(\lambda+\theta+s)^3\right\}}$$

$$=Q_{23}^{**}(s)=Q_{43}^{**}(s) \quad (7)$$

$$Q_{34}^{**}(s)=\frac{\lambda^2(\lambda+s+1)}{\left\{(\lambda+1)(\lambda+s)^2\right\}} \quad (8)$$

Taking $s \to 0$, we get the following transition probabilities

$$p_{01} = p_{10} + p_{12} = p_{21} + p_{23} = p_{34} = p_{41} + p_{43} = 1$$

Mean Sojourn Times

The expected time taken by the system in a particular state before transiting to any other state is known as mean sojourn time or mean survival time in the state. If T_i be the sojourn time in the state i, then the mean sojourn time in the state i is

$$\mu_i=\int_0^\infty Pr(T_i>t) \text{ or } \mu_i=\Sigma_j m_{ij}.$$

where,

$$m_{ij}=-\frac{d}{ds}\left[Q_{ij}^{**}(s)\right]_{s=0}$$

We have

$$m_{01}=-\frac{d}{ds}\left[Q_{01}^{**}(s)\right]_{s=0}=\frac{2+\theta}{\theta(\theta+1)} \quad (9)$$

$$m_{10}=\frac{\lambda^2\left\{(\lambda+\theta)^2(1+\theta)+2(\lambda+\theta)(1+2\theta)+6\theta\right\}}{\left\{(\lambda+1)(\theta+1)+(\lambda+\theta)^4\right\}}$$

$$= m_{21} = m_{41} \quad (10)$$

$$m_{10} = \frac{\theta^2\{(\lambda+\theta)^2(1+\lambda)+2(\lambda+\theta)(1+2\lambda)+6\lambda\}}{\{(\lambda+1)(\theta+1)+(\lambda+\theta)^4\}}$$

$$= m_{23} = m_{43} \quad (11)$$

$$m_{01} = \frac{2+\lambda}{\lambda(\lambda+1)} \quad (12)$$

Reliability Measures

The following reliability measures have been evaluated for the system model

Mean Time to System Failure (MTSF)

Let $\phi_i(t)$ be the c.d.f. of first passage time from regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$

$$\phi_0(t) = Q_{01}(t) ® \phi_1(t) \quad (13)$$

$$\phi_1(t) = Q_{10}(t) ® \phi_0(t) + Q_{13.2}(t) \quad (14)$$

Taking Laplace Stieltjes Transform, we get

$$\phi_0^{**}(s) - \phi_{01}^{**}(s)\phi_1^{**}(s) = 0 \quad (15)$$

$$\phi_1^{**}(s) - \phi_{10}^{**}(s)\phi_0^{**}(s) = \phi_{13.2}^{**}(s) \quad (16)$$

Using Cramer's Rule for solving the above equations to obtain $\phi_0^{**}(s)$, we get

$$\begin{vmatrix} 1 & -\phi_{01}^{**}(s) \\ -\phi_{10}^{**}(s) & 1 \end{vmatrix} \begin{vmatrix} \phi_0^{**}(s) \\ \phi_1^{**}(s) \end{vmatrix} = \begin{vmatrix} 0 \\ \phi_{13.2}^{**}(s) \end{vmatrix}$$

$$D = 1 - \phi_{01}^{**}(s)\phi_{10}^{**}(s)$$

$$N = \phi_{01}^{**}(s)\phi_{13.2}^{**}(s)$$

On solving for $\phi_0^{**}(s)$, we have

$$\phi_0^{**}(s) = \frac{N}{D} = \frac{\phi_{01}^{**}(s)\,\phi_{13.2}^{**}(s)}{1-\phi_{01}^{**}(s)\phi_{10}^{**}(s)} \quad (17)$$

Also,

$$\text{MTSF} = \lim_{S\to 0}\frac{\phi_{01}^{**}(s)}{S}$$

$$= \lim_{S\to 0}\frac{1-\phi_{01}^{**}(s)\phi_{10}^{**}(s)-\phi_{01}^{**}(s)\phi_{13.2}^{**}(s)}{S[1-\phi_{01}^{**}(s)\phi_{10}^{**}(s)]} = \left(\frac{0}{0}\right) \quad (18)$$

So applying L' Hospital's Rule, we have

$$\lim_{S\to 0}\frac{-\phi_{01}^{**\prime}(s)[\phi_{10}^{**}(s)+\phi_{13.2}^{**}(s)]-\phi_{01}^{**}(s)[\phi_{01}^{**\prime}(s)+\phi_{13.2}^{**\prime}(s)]}{[1-\phi_{01}^{**}(s)\phi_{10}^{**}(s)]+S[1-\phi_{01}^{**\prime}(s)\phi_{10}^{**}(s)-\phi_{01}^{**}(s)\phi_{10}^{**\prime}(s)]}$$

$$= \frac{m_{01}p_{01}+m_{01}p_{13.2}+m_{10}p_{01}+m_{13.2}p_{01}}{1-p_{01}p_{10}}$$

$$MTSF = \frac{\mu_1'}{1-p_{01}p_{10}}$$

where, $\mu_1' = m_0 + m_{12}p_{23} + m_{23}p_{12}$ (19)

Reliability

Let $\phi_i(t)$ be the c.d.f. of first passage time from regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$

$$R^*(S) = \frac{1-\phi_0^{**}(s)}{S} \text{ and } R(t) = L^{-1}(R^*(S))$$

Availability

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state S_i at $t = 0$. The recursive relations for $A_i(t)$ are given as

$$A_0(t) = M_0(t) + q_{01}(t)©A_1(t)$$

$$A_1(t) = M_1(t) + q_{10}(t)©A_0(t) + q_{11.2}(t)©A_1(t) + q_{11.2(34)^n}(t)©A_1(t)$$

Taking Laplace transform of above equations, we have

$$A_0^*(s) = M_0^*(s) + q_{01}^*(s)A_1^*(s)$$

$$A_1^*(s) = M_1^*(s) + q_{10}^*(s)A_0^*(s) + Q_{11.2}^*(s)A_1^*(s) + Q_{11.2(34)^n}^*(s)A_1^*(s)$$

$$A_0^*(s) - q_{01}^*(s)A_1^*(s) = M_0^*(s)$$

$$A_1^*(s)(1-q_{11.2}^*(s)-q_{11.2(34)^n}^*(s)) - q_{10}^*(s)A_0^*(s) = M_1^*(s)$$

Using Cramer's Rule for solving the above equations to obtain $A_0^*(s)$, we get

$$\begin{vmatrix} 1 & -q_{01}^{**}(s) \\ -q_{10}^*(s) & 1-q_{11.2}^*(s)-q_{11.2(34)^n}^*(s) \end{vmatrix} \begin{vmatrix} A_0^*(s) \\ A_1^*(s) \end{vmatrix} = \begin{vmatrix} M_0^*(s) \\ M_1^*(s) \end{vmatrix}$$

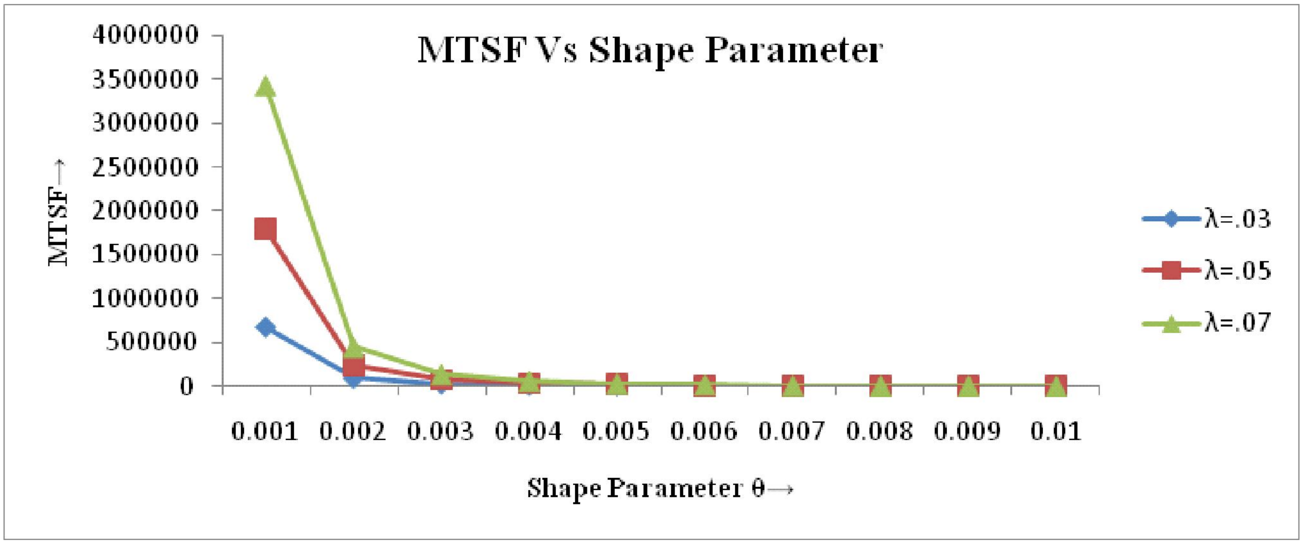

Fig. 2 : MTSF cs Shape Parameter.

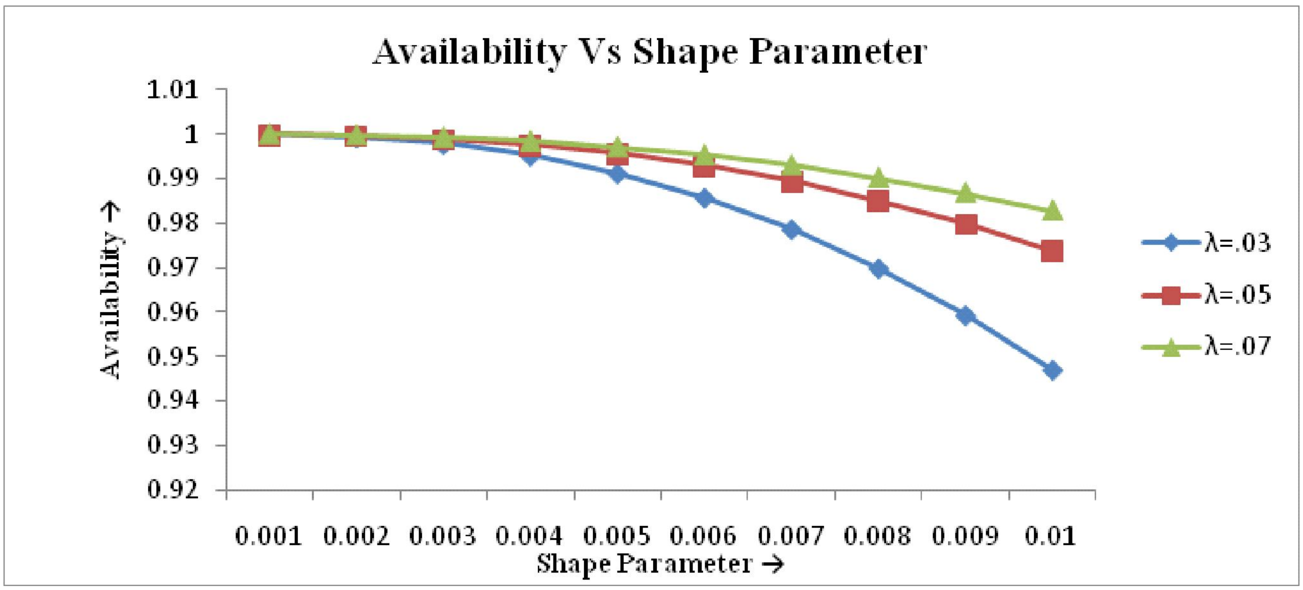

Fig. 3 : Availability vs Shape Parameter.

$$D_1 = 1 - q^*_{11.2}(s) - q^*_{11.2(34)^n}(s) - q^*_{01}(s)q^*_{10}(s)$$

$$N_1 = M^*_0(s)\left(1 - q^{**}_{11.2}(s) - q^{**}_{11.2(34)^n}(s)\right) + M^*_1(s)q^*_{01}(S)$$

$$A^*_0(s) = \frac{N_1}{D_1} = \frac{M^*_0(s)\left(1 - q^*_{11.2}(s) - q^*_{11.2(34)^n}(s)\right) + M^*_1(s)q^*_{01}(s)}{1 - q^*_{11.2}(s) - q^*_{11.2(34)^n}(s) - q^*_{01}(s)q^*_{10}(s)}$$

$$A(\infty) = \lim_{S\to 0} A^*_0(s) = \frac{sN_1}{D_1} = \left(\frac{0}{0}\right) \text{ form}$$

Now using L-Hospital's Rule, we get

$$\lim_{s\to 0}\frac{\left\{M^*_0(s)\left(1 - q^*_{11.2}(s) - q^*_{11.2(34)^n}(s)\right) + M^*_1(S)q^*_{01}(s)\right\} + \left\{\begin{matrix} M^{*}_0{}'(s)\left(1 - q^*_{11.2}(s) - q^*_{11.2(34)^n}(s)\right) \\ + M^{*}_0{}'(s)\left(-q^{*'}_{11.2}(s) - q^{*'}_{11.2(34)^n}(s)\right) \\ + M^{*}_1{}'(S)q^{**}_{01}(s) + M^*_1(S)q^{*'}_{01}(s) \end{matrix}\right\}}{-q^{*'}_{11.2}(s) - q^{*'}_{11.2(34)^n}(s) - q^{*'}_{01}(s)q^*_{10}(s) - q^*_{01}(s)q^{*'}_{01}(s)}$$

Hence, $A(\infty) = \dfrac{\mu_0 p_{10} + \mu_1}{\mu'_1 + \mu_0 p_{10}}$

Table 1 : MTSF vs Shape Parameter.

Shape Parameter θ	MTSF		
	λ=.03	λ=.05	λ=.07
0.001	675610.3	1788934	3432516
0.002	94654.26	239807.1	451160.8
0.003	31272.95	76063.32	140439.1
0.004	14637.6	34289.98	62189.55
0.005	8275.561	18727.59	33392.73
0.006	5265.204	11541.16	20248.9
0.007	3630.914	7727.277	13350.51
0.008	2654.199	5495.64	9356.64
0.009	2027.601	4091.888	6869.736
0.01	1603.152	3158.303	5231.715

Table 2 : Availability vs Shape Parameter.

Shape Parameter θ	Availability		
	λ=.03	λ=.05	λ=.07
0.001	0.999902	0.999949	0.999966
0.002	0.999289	0.999634	0.999757
0.003	0.997797	0.998874	0.999255
0.004	0.995159	0.997542	0.998379
0.005	0.991174	0.995545	0.997073
0.006	0.985682	0.992812	0.995292
0.007	0.978566	0.989286	0.993003
0.008	0.969739	0.984921	0.990177
0.009	0.959152	0.979681	0.986791
0.01	0.946788	0.97354	0.982827

where, $\mu'_1 = \mu_1 + \mu_2 p_{12} + \dfrac{p_{12} p_{23(\mu_3+\mu_4)}}{p_{41}}$

Numerical and Graphical Representation of MTSF and Availability

Numerical and Graphical Representation of MTSF and Availability are given in Tables 1 & 2 and Figs. 2 and 3.

6. Conclusion and Discussion

A cold standby system of three identical units is considered to know the effect of standby units on availability. The system model is developed by assuming Lindley distribution for failure and repair times. The results for availability are obtained by using arbitrary values of the parameters. The behaviour of mean time to system failure (MTSF) and availability has been observed for arbitrary values of failure and repair rates as shown in Figs. 2 and 3, respectively. It is found that MTSF declines sharply with the increase of shape parameter (q) while it increases with the increasing of repair rate (l). And, availability of the system goes on decreasing with the increase of shape parameter. However, it can be increased by increasing the repair rate. The results for these reliability measures are also given in the respective Tables 1 and 2.

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