Int. J. Agricult. Stat. Sci. Vol. 15, No. 1, pp. 289-302, 2019 ***ISSN : 0973-1903***

ORIGINAL ARTICLE

RELATIONSHIPS FOR MOMENTS OF GENERALIZED ORDER STATISTICS FROM SCHABE DISTRIBUTION AND RELATED INFERENCE

Jagdish Saran, Narinder Pushkarna and Kanika Verma[1]*

Department of Statistics, Faculty of Mathematical Sciences, University of Delhi, Delhi - 110 007, India.
E-mail : kanikaverma29apr@gmail.com

Abstract : In this paper some recurrence relations satisfied by single and product moments of generalized order statistics from Schabe distribution have been obtained. Then, we use these results to compute the first two moments of order statistics for some specific values of the parameters. Further, we use the results on order statistics to obtain BLUEs of location and scale parameters based on type-II right censored samples. In addition, we carry out simulation study through Monte Carlo simulation to show the usefulness of the findings.

Key words : Generalized order statistics, Schabe distribution, Single moments, Product moments, Recurrence relations, Best linear unbiased estimators, Type-II right censored samples.

Mathematics Subject Classification : 62G30

1. Introduction

Let $\{X_n, n \geq 1\}$ be a sequence of independent and identically distributed random variables with cumulative distribution function (cdf) $F(x)$ and probability density function (pdf) $f(x)$.

Assume that $k > 0$, $n \in N$, $n \geq 2$, $\tilde{m} = (m_1, m_2, ..., m_{n-1}) \in R^{n-1}$, $M_r = \sum_{j=r}^{n-1} m_j$ such that $\gamma_r = k + (n-r)+M_r > 0$ for all $r \in \{1, 2, ..., n-1\}$. Then $X(r,n,\tilde{m},k)$, $r = 1, 2, ..., n$, are called generalized order statistics if their joint pdf is given by

$$f_{X(1,n,\tilde{m},k),...,X(n,n,\tilde{m},k)}(x_1,...,x_n) = k\left(\prod_{j=1}^{n-1}\gamma_j\right)$$

$$\times\left(\prod_{i=1}^{n-1}(1-F(x_n))^{m_i} f(x_i)\right)$$

$$\times(1-F(x_n))^{k-1} f(x_n),$$

$$F^{-1}(0+) < x_1 \leq ... \leq x_n < F^{-1}(1). \quad (1)$$

Choosing the parameters appropriately, models such as ordinary order statistics (γ_i = n – i + 1; i = 1, 2, ..., n, *i.e.* $m_1 = m_2 = ... = m_{n-1} = 0$, $k = 1$), kth record values ($\gamma_i = k$, *i.e.*, $m_1 = m_2 = ... = m_{n-1} = -1$, $k \in N$), sequential order statistics ($\gamma_i = (n - i + 1)\alpha_i$; $\alpha_1, \alpha_2, ..., \alpha_n > 0$), order statistics with non-integral sample size ($\gamma_i = \alpha - i + 1$, $\alpha > 0$), Pfeifer's record values ($\gamma_i = \beta_i$; $\beta_1, \beta_2, .., \beta_n > 0$) and progressively type-II right censored order statistics ($m_i \in N_0$, $k \in N$) are obtained [Kamps (1995a, b), Kamps and Cramer (2001)].

The joint pdf of first r generalized order statistics is given by

$$f_{X(1,n,\tilde{m},k),...,X(r,n,\tilde{m},k)}(x_1,...,x_r) = c_{r-1}\left(\prod_{i=1}^{r-1}(1-F(x_i))^{m_i} f(x_r)\right)$$

$$\times(1-F(x_r))^{k+(n-r)+M_r-1} f(x_r),$$

$$F^{-1}(0+) < x_1 \leq ... \leq x_r < F^{-1}(1). \quad (2)$$

We may consider two cases here :

Case 1 : $m_1 = m_2 = ... = m_{n-1} = m$.

Case 2 : $\gamma_i \neq \gamma_j$; $i \neq j$, $i, j = 1, 2, ..., n-1$.

For Case 1, the rth generalized order statistic will

Author for correspondence** ***Received *December 24, 2018* ***Revised*** *February 24, 2019* ***Accepted*** *March 18, 2019*

be denoted by $X(r, n, m, k)$. The pdf of $X(r, n, m, k)$ is given by

$$f_{X(r,n,m,k)}(x) = \frac{c_{r-1}}{(r-1)!}(1-F(x))^{\gamma_r - 1} f(x) g_m^{r-1}(F(x)),$$
$$x \in R \tag{3}$$

and the joint pdf of $X(r, n, m, k)$ and $X(s, n, m, k)$, $1 \le r < s \le n$, is given by

$$f_{X(r,n,m,k),X(s,n,m,k)}(x,y) = \frac{c_{s-1}}{(r-1)!(s-r-1)!}$$
$$\times [1-F(x)]^m f(x) g_m^{r-1}(F(x))$$
$$\times [h_m(F(y)) - h_m(F(x))]^{s-r-1} [1-(F(y))]^{\lambda_s - 1} f(y), x < y, \tag{4}$$

where,

$$c_{r-1} = \prod_{j=1}^{r} \gamma_j, \ \gamma_j = k + (n-j)(m+1),$$
$$r = 1, 2, ..., n,$$
$$g_m(x) = h_m(x) - h_m(0), \ x \in [0,1),$$
$$h_m(x) = \begin{cases} \frac{-1}{m+1}(1-x)^{m+1}, & m \ne -1 \\ -log(1-x), & m = -1 \end{cases}$$

[Kamps (1995a, b)].

For case 2, $X(r,n,\tilde{m},k)$ denotes the rth generalized order statistic. The pdf of $X(r,n,\tilde{m},k)$ is given by

$$f_{X(r,n,\tilde{m},k)}(x) = c_{r-1} f(x) \sum_{i=1}^{r} a_i(r)(1-F(x))^{\gamma_i - 1},$$
$$x \in R, \tag{5}$$

and the joint pdf of $X(r,n,\tilde{m},k)$ and $X(s,n,\tilde{m},k)$, $1 \le r < s \le n$, is given by

$$f_{X(r,n,\tilde{m},k),X(s,n,\tilde{m},k)}(x,y)$$
$$= c_{s-1} \left\{ \sum_{i=r+1}^{s} a_i^{(r)}(s) \left(\frac{1-F(y)}{1-F(x)} \right)^{\gamma_i} \right\}$$
$$\times \left\{ \sum_{i=1}^{r} a_i(r)(1-F(x))^{\gamma_i} \right\}$$
$$\times \frac{f(x)}{1-F(x)} \frac{f(y)}{1-F(y)}, \ x < y, \tag{6}$$

where,

$$c_{r-1} = \prod_{i=1}^{r} \gamma_i, \ \gamma_i = k + n - i + M_i, \ r = 1, 2, ..., n,$$

$$a_i(r) = \prod_{j(\ne i)=1}^{r} \frac{1}{(\gamma_j - \gamma_i)}, \quad 1 \le i \le r \le n$$

and

$$a_i^{(r)}(s) = \prod_{j(\ne i)=r+1}^{s} \frac{1}{(\gamma_j - \gamma_i)}, \ r+1 \le i \le s \le n$$

[Kamps and Cramer (2001)].

Further, it can be easily proved that

$$\left. \begin{aligned} a_i(r) &= (\gamma_{r+1} - \gamma_i) a_i(r+1), \\ c_{r-1} &= \frac{c_r}{\gamma_{r+1}}, \\ \sum_{i=1}^{r+1} a_i(r+1) &= 0, \\ \sum_{i=r+1}^{s} a_i^{(r)}(s) &= 0. \end{aligned} \right\} \tag{7}$$

Also, for $m_i = m_j = m$, it can be shown that

$$\sum_{i=1}^{r} a_i(r)(1-F(x))^{\gamma_i} = \frac{(1-F(x))^{\gamma_r}}{(r-1)!} g_m^{r-1}(F(x)). \tag{8}$$

and

$$\sum_{i=r+1}^{s} a_i^{(r)}(s) \left(\frac{1-F(y)}{1-F(x)} \right)^{\gamma_i} = \frac{1}{(s-r-1)!} \left(\frac{1-F(y)}{1-F(x)} \right)^{\gamma_s}$$
$$\times \left(\frac{1}{1-F(x)} \right)^{(m+1)(s-r-1)} [h_m(F(y)) - h_m(F(x))]^{s-r-1}. \tag{9}$$

Several authors like Kamps and Gather (1997), Keseling (1999), Cramer and Kamps (2000), Ahsanullah (2000), Pawlas and Szynal (2001), Ahmad and Fawzy (2003), Athar and Islam (2004), Ahmad (2007), Khan *et al*. (2007), Khan *et al*. (2010) and Saran and Pandey (2004, 2009) have done some work on generalized order statistics. In this paper, in Section 3, we have established recurrence relations for single and product moments of generalized order statistics from Schabe

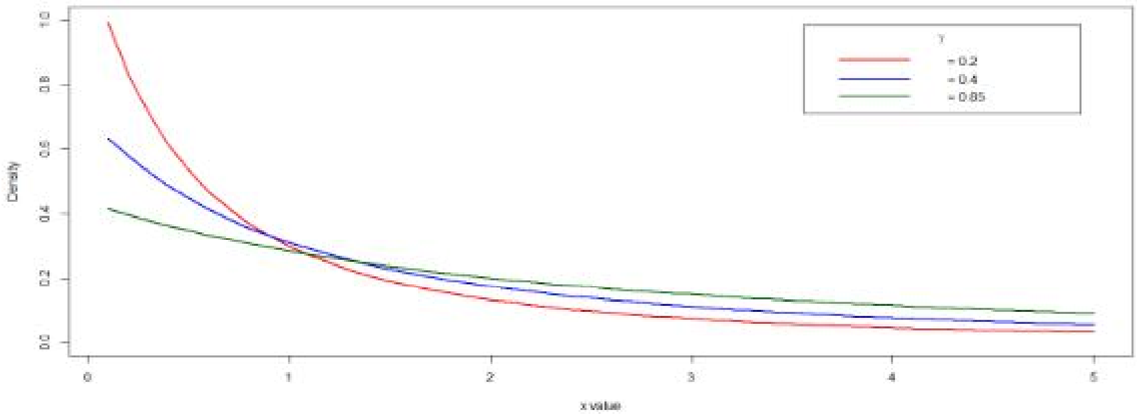

Fig. 1 : Probability density function of Schabe distribution with θ = 5.

distribution for Case 2 only, *i.e.*, for $\gamma_i \neq \gamma_j$; $i \neq j$, $i, j = 1, 2, \ldots, n-1$. Then we use these results to compute means and variances of order statistics for some specific values of the parameters. Further, in Section 4, we use the results on order statistics to obtain BLUEs of location and scale parameters based on type-II right censored samples. In Section 5, we carry out simulation study through Monte Carlo simulation to show the usefulness of the findings.

Regarding the best linear unbiased estimation, we may mention, for example, Jabeen *et al*. (2013), who have discussed the estimation of location and scale parameters of Weibull distribution using the concept of generalized order statistics under type-II censored data. Mahmoud *et al*. (2003) have obtained the best linear unbiased estimates (BLUEs) of future order statistics from inverse Weibull distribution by using type-II censoring. Samuel and Thomas (1997) have derived the single and product moments of order statistics from U-shaped distribution. Then, they have used these moments to obtain the BLUEs for the location and scale parameters. Balakrishnan and Chandramouleeswaran (1996) have obtained the BLUEs of future order statistics from Laplace distribution based on type-II censored samples.

2. Schabe Distribution

Schabe distribution is a lifetime distribution with bathtub shaped failure rate and is extremely useful in reliability. It is a reparameterization of Pareto distribution which has decreasing failure rate. Schabe (1994) gave a method to construct bathtub shaped failure rate distribution from a distribution with decreasing failure rate by the method of truncation described below.

Let $F(x)$ be a twice differentiable function with decreasing failure rate $\lambda(x)$ and support on $[0, \infty)$. Let θ be a truncation point with $0 < \theta < \infty$. If

$$G(x) = \begin{cases} F(x)/F(\theta), & x \le \theta \\ 1, & otherwise \end{cases}$$

then $G(x)$ has bathtub shaped failure rate if

$$\lambda'(x)(F(\theta) - F(x)) + (\lambda(x))^2(1 - F(x))$$

has one and only one zero on $[0, \infty)$, [Schabe (1994)].

Now, consider Pareto distribution defined by

$$F(x) = \frac{1}{\left(1 + \frac{T}{x}\right)^{\alpha}}, \quad x \ge 0.$$

which has decreasing failure rate. In order to keep the model simple, the shape parameter is set to $\alpha = 1$. Then

$$G(x) = \begin{cases} \frac{x(T+\theta)}{\theta(T+x)}, & x \le \theta \\ 1, & otherwise \end{cases}$$

The model can be reparameterized by setting $\gamma = \frac{T}{\theta}$. Hence, we get Schabe distribution with probability density function (pdf) of the form

$$f(x) = \frac{(1+\gamma)\gamma\theta}{(x+\theta\gamma)^2}, \; 0 < x \le \theta, \; 0 < \gamma < 1, \; \theta > 0, \quad (10)$$

and the cumulative distribution function (cdf) of the form

$$F(x) = \frac{(1+\gamma)x}{(x+\theta\gamma)}. \quad (11)$$

It may be noted that the characterizing differential equation of Schabe distribution (10) is given by

$$f(x)\left[x\theta - x^2 + \theta^2\gamma - x\theta\gamma\right] = (\gamma+1)\theta(1-F(x)) \tag{12}$$

More details on this distribution can be found in Schabe (1994). The graph of the density function of Schabe distribution (10) for θ = 5 and for different values of γ = 0.20, 0.40, 0.85 is provided in Fig. 1.

The cdf of the location-scale parameter Schabe distribution is given by

$$F(x) = \frac{(1+\gamma)\dfrac{(x-\mu)}{\sigma}}{\left(\dfrac{(x-\mu)}{\sigma} + \theta\gamma\right)}, \ \mu < x \le \mu + \theta\sigma, \ \mu \ge 0,$$

$$0 < \gamma < 1, \ \theta > 0. \tag{13}$$

3. Recurrence Relations for Single and Product Moments of Generalized order statistics from Schabe Distribution for Case 2

Theorem 3.1: For the distribution given in (10) and $n \in N$, $r \ge 2$, $k \ge 1$, we have

$$E\left[X^{j+2}(r,n,\tilde{m},k)\right]$$

$$= \left(\theta - \theta\gamma - \frac{(1+\gamma)\theta\gamma_r}{j+1}\right) E\left[X^{j+1}(r,n,\tilde{m},k)\right]$$

$$-\theta^2\gamma E\left[X^{j}(r,n,\tilde{m},k)\right]$$

$$+\theta(1+\gamma)\gamma_r E\left[X^{j+1}(r-1,n,\tilde{m},k)\right] \tag{14}$$

and for r=1,

$$E\left[X^{j+2}(1,n,\tilde{m},k)\right]$$

$$= \left(\theta - \theta\gamma - \frac{(1+\gamma)\theta\gamma_1}{j+1}\right) E\left[X^{j+1}(1,n,\tilde{m},k)\right]$$

$$-\theta^2\gamma E\left[X^{j}(1,n,\tilde{m},k)\right]. \tag{15}$$

Proof : From (5) and (12), we have

$$-E\left[X^{j+2}(r,n,\tilde{m},k)\right] + (\theta-\gamma\theta)E\left[X^{j+1}(r,n,\tilde{m},k)\right]$$

$$+\theta^2\gamma E\left[X^{j}(r,n,\tilde{m},k)\right]$$

$$= c_{r-1}\int_0^\theta \left\{-x^{j+2} + (\theta-\theta\gamma)x^{j+1}\right.$$

$$\left.+\theta^2\gamma x^j\right\} f(x)\sum_{i=1}^{r} a_i(r)\left[1-F(x)\right]^{\gamma_i - 1} dx$$

$$= c_{r-1}\int_0^\theta x^j\left\{-x^2 + (\theta-\theta\gamma)x^1\right.$$

$$\left.+\theta^2\gamma\right\} f(x)\sum_{i=1}^{r} a_i(r)\left[1-F(x)\right]^{\gamma_i - 1} dx$$

$$= c_{r-1}(1+\gamma)\theta\int_0^\theta x^j \sum_{i=1}^{r} a_i(r)\left[1-F(x)\right]^{\gamma_i} dx$$

$$= c_{r-1}(1+\gamma)\theta\left[\frac{x^{j+1}}{j+1}\sum_{i=1}^{r} a_i(r)\left[1-F(x)\right]^{\gamma_i}\Big|_0^\theta + \right.$$

$$\left.\int_0^\theta \frac{x^{j+1}}{j+1}\sum_{i=1}^{r} a_i(r)\gamma_i\left[1-F(x)\right]^{\gamma_i - 1} f(x)dx\right]$$

$$= \frac{(1+\gamma)\theta c_{r-1}}{j+1}\int_0^\theta x^{j+1}\sum_{i=1}^{r} a_i(r)\gamma_i\left[1-F(x)\right]^{\gamma_i - 1} f(x)dx$$

$$= \frac{(1+\gamma)\theta c_{r-1}}{j+1}\int_0^\theta x^{j+1}\sum_{i=1}^{r-1} a_i(r)\gamma_i\left[1-F(x)\right]^{\gamma_i - 1} f(x)dx$$

$$+ \frac{(1+\gamma)\theta c_{r-1}}{j+1}\int_0^\theta x^{j+1} a_r(r)\gamma_r\left[1-F(x)\right]^{\gamma_r - 1} f(x)dx$$

$$= -\frac{(1+\gamma)\theta c_{r-1}}{j+1}\int_0^\theta x^{j+1}\sum_{i=1}^{r-1} a_i(r)\left\{(\gamma_r-\gamma_i)-\gamma_r\right\}$$

$$\times\left[1-F(x)\right]^{\gamma_i - 1} f(x)dx$$

$$+ \frac{(1+\gamma)\theta c_{r-1}}{j+1}\int_0^\theta x^{j+1} a_r(r)\gamma_r\left[1-F(x)\right]^{\gamma_r - 1} f(x)dx$$

$$= -\frac{(1+\gamma)\theta c_{r-1}}{j+1}\int_0^\theta x^{j+1}\sum_{i=1}^{r-1} a_i(r-1)$$

$$\times\left[1-F(x)\right]^{\gamma_i - 1} f(x)dx$$

$$+ \gamma_r\frac{(1+\gamma)\theta c_{r-1}}{j+1}\int_0^\theta x^{j+1}\sum_{i=1}^{r} a_i(r)\left[1-F(x)\right]^{\gamma_i - 1} f(x)dx$$

$$= \frac{\gamma_r(1+\gamma)\theta}{j+1}\left[E\left[X^{j+1}(r,n,\tilde{m},k)\right]\right.$$

$$\left. - E\left[X^{j+1}(r-1,n,\tilde{m},k)\right]\right],$$

and after rearranging terms in the above expression, we get the required result as given in (14). Proceeding in a similar manner, the result given in (15) can easily be established. It may be noted that the relation (15) follows from (14) by taking therein $X(0,n,\tilde{m},k) = 0$.

Remark 3.1 : Putting $m_i = m_j = m$ in (5) and using

(8), the recurrence relations established in Theorem 3.1 reduce to the recurrence relations for single moments of generalized order statistics from Schabe distribution for Case 1, *i.e.*, when $m_1 = m_2 = \ldots = m_{n-1} = m$.

Theorem 3.2: For the distribution given in (10) and $n \in N$, $r \geq 2$, $k \geq 1$, we have

$$E\left[X^j(r-1,n,m,k)\right]-E\left[X^j(r-1,n-1,m^*,k)\right]$$

$$=-\frac{(m+1)c_{r-2}^{(n)}}{\gamma_1(r-2)!}\int_0^\theta jx^{j-1}[1-F(x)]^{\gamma_r}g_m^{r-1}(F(x))dx, \tag{16}$$

where, $m^* = (m_2 = m_3 = \ldots = m_{n-1}) \in R$ and

$$c_{r-2}^{(n)}=\prod_{i=1}^{r-1}[k+(n-i)(m+1)].$$

Proof : Noting that $\gamma_1 c_{r-2}^{(n-1)} = \gamma_r c_{r-2}^{(n)}$ and $(m+1)g_m(F(x))+[1-F(x)]^{m+1} = 1$, the L.H.S. of (16) is given by

$$E\left[X^j(r-1,n,m,k)\right]-E\left[X^j(r-1,n-1,m^*,k)\right]$$

$$=E\left[X^j(r-1,n,m,k)\right]$$

$$-\frac{c_{r-2}^{(n-1)}}{(r-2)!}\int_0^\theta x^j[1-F(x)]^{\gamma_r-1}f(x)g_m^{r-2}(F(x))$$

$$\times\left\{(1-F(x))^{\gamma_{r-1}-\gamma_r}+(m+1)g_m(F(x))\right\}dx$$

$$=E\left[X^j(r-1,n,m,k)\right]-\frac{(m+1)c_{r-2}^{(n-1)}}{(r-2)!}$$

$$\times\int_0^\theta x^j[1-F(x)]^{\gamma_{r-1}^{(n-1)}-1}f(x)g_m^{r-1}(F(x))dx$$

$$-\frac{c_{r-2}^{(n-1)}}{(r-2)!}\int_0^\theta x^j[1-F(x)]^{\gamma_{r-1}-1}f(x)g_m^{r-2}(F(x))dx$$

$$=-\frac{(m+1)c_{r-2}^{(n-1)}}{(r-2)!}\int_0^\theta x^j[1-F(x)]^{\gamma_{r-1}^{(n-1)}-1}g_m^{r-1}(F(x))f(x)dx$$

$$+\left\{1-\frac{\gamma_r}{\gamma_1}\right\}\frac{c_{r-2}^{(n)}}{(r-2)!}\int_0^\theta x^j[1-F(x)]^{\gamma_{r-1}-1}f(x)g_m^{r-2}(F(x))dx$$

$$=-\frac{(m+1)c_{r-2}^{(n)}\gamma_r}{\gamma_1(r-2)!}\int_0^\theta x^j[1-F(x)]^{\gamma_r-1}g_m^{r-1}(F(x))f(x)dx$$

$$+\frac{(m+1)c_{r-2}^{(n)}(r-1)}{\gamma_1(r-2)!}\int_0^\theta x^j[1-F(x)]^{\gamma_{r-1}-1}g_m^{r-2}(F(x))f(x)dx$$

$$=\frac{(m+1)c_{r-2}^{(n)}}{\gamma_1(r-2)!}\int_0^\theta\left\{-\gamma_r[1-F(x)]^{\gamma r-1}g_m^{r-1}(F(x))f(x)x^j\right.$$

$$\left.+(r-1)[1-F(x)]^{\gamma_r+m}g_m^{r-2}(F(x))f(x)x^j\right\}dx.$$

Integrating by parts the first integral on the R.H.S. of the above equation by taking $x^j g_m^{r-1}(F(x))$ for differentiation and the rest of the integrand for integration and then after some simplification, it leads to the required result (16).

Remark 3.2 : If we take k=1 and $m_i = m_j = m = 0$ in (14), (15) and (16), we get the recurrence relations for single moments of order statistics from Schabe distribution. Numerical computations for the means and variances of order statistics from Schabe distribution for arbitrarily chosen values of θ and γ, *viz.* (θ = 5, γ = 0.4), (θ = 5, γ = 0.85) and (θ = 6, γ = 0.85) and for sample sizes n = 5(1)10 are given in Tables 1 and 2.

Theorem 3.3 : For $1 \leq r < s \leq n$ and $i, j \geq 0$,

$$E\left[X^i(r,n,\tilde{m},k)X^{j+2}(s,n,\tilde{m},k)\right]$$

$$=\left(\theta-\theta\gamma-\frac{(1+\gamma)\theta\gamma_s}{j+1}\right)E\left[X^i(r,n,\tilde{m},k)X^{j+1}(s,n,\tilde{m},k)\right]$$

$$-\theta^2\gamma E\left[X^i(r,n,\tilde{m},k)X^j(s,n,\tilde{m},k)\right]$$

$$+\theta(1+\gamma)\gamma_s E\left[X^i(r,n,\tilde{m},k)X^{j+1}(s-1,n,\tilde{m},k)\right]. \tag{17}$$

Proof : Using (6) and (12), we have

$$-E\left[X^i(r,n,\tilde{m},k)X^{j+2}(s,n,\tilde{m},k)\right]$$

$$+(\theta-\theta\gamma)E\left[X^i(r,n,\tilde{m},k)X^{j+1}(s,n,\tilde{m},k)\right]$$

$$+\theta^2\gamma E\left[X^i(r,n,\tilde{m},k)X^j(s,n,\tilde{m},k)\right]$$

$$=c_{s-1}\int_0^\theta\int_x^\theta x^i y^j\left\{-y^2+(\theta-\theta\gamma)y^1+\theta^2\gamma\right\}$$

$$\times\left\{\sum_{i=1}^r a_i(r)[1-F(x)]^{\gamma_i}\right\}$$

$$\times\sum_{i=r+1}^s a_i^{(r)}(s)\left[\frac{1-F(y)}{1-F(x)}\right]^{\gamma_i}\frac{f(x)}{1-F(x)}\frac{f(y)}{1-F(y)}dydx$$

$$=(1+\gamma)\theta c_{s-1}\int_0^\theta x^i\left\{\sum_{i=1}^r a_i(r)[1-F(x)]^{\gamma_i}\right\}$$

$$\times\frac{f(x)}{1-F(x)}\int_x^\theta y^j\sum_{i=r+1}^r a_i^{(r)}(s)$$

Table 1 : Means and variances of order statistics from Schabe distribution.

		θ = 5, γ = 0.4		θ = 5, γ = 0.85	
r	*n*	$E(X_{r:n})$	$\mathrm{Var}(X_{r:n})$	$E(X_{r:n})$	$\mathrm{Var}(X_{r:n})$
1	5	0.306324	0.103811	0.457999	0.201299
2	5	0.711470	0.284124	1.012718	0.459337
3	5	1.264783	0.575093	1.693775	0.752128
4	5	2.048839	0.971993	2.542366	0.994205
5	5	3.207266	1.226258	3.616654	0.945147
1	6	0.252965	0.070386	0.382841	0.142867
2	6	0.573118	0.185521	0.833785	0.324001
3	6	0.988173	0.366484	1.370583	0.537908
4	6	1.541393	0.630675	2.016966	0.757441
5	6	2.302562	0.949525	2.805066	0.905553
6	6	3.388206	1.085167	3.778971	0.794984
1	7	0.215283	0.050628	0.328682	0.106301
2	7	0.479054	0.129299	0.707796	0.239067
3	7	0.808279	0.248656	1.148758	0.397444
4	7	1.228033	0.422906	1.666350	0.572108
5	7	1.776413	0.657622	2.279928	0.735094
6	7	2.513022	0.911260	3.015121	0.819305
7	7	3.534071	0.965217	3.906279	0.677479
1	8	0.187299	0.038065	0.287852	0.082020
2	8	0.411171	0.094717	0.614498	0.182905
3	8	0.682702	0.177746	0.987690	0.303101
4	8	1.017573	0.296753	1.417204	0.439379
5	8	1.438493	0.460472	1.915496	0.580689
6	8	1.979165	0.666289	2.498587	0.700239
7	8	2.690974	0.866249	3.187300	0.740413
8	8	3.654513	0.863304	4.008991	0.584091
1	9	0.165715	0.029615	0.255992	0.065127
2	9	0.359969	0.072124	0.542727	0.144089
3	9	0.590379	0.132504	0.865696	0.237630
4	9	0.867347	0.217089	1.231677	0.344750
5	9	1.205356	0.332861	1.649113	0.460858
6	9	1.625002	0.484294	2.128603	0.574370
7	9	2.156246	0.663214	2.683579	0.660508
8	9	2.843754	0.819222	3.331220	0.670034
9	9	3.755858	0.776378	4.093712	0.508749
1	10	0.148571	0.023674	0.230452	0.052923
2	10	0.320016	0.056631	0.485852	0.116256
3	10	0.519783	0.102169	0.770227	0.190727
4	10	0.755102	0.164521	1.088457	0.276180
5	10	1.035715	0.248695	1.446508	0.370684
6	10	1.374998	0.359470	1.851718	0.468935
7	10	1.791672	0.498062	2.313193	0.559477
8	10	2.312493	0.652617	2.842316	0.619815
9	10	2.976569	0.772673	3.453446	0.607893
10	10	3.842445	0.701815	4.164853	0.447123

Table 2 : Means and variances of order statistics from Schabe distribution.

θ = 6, γ = 0.85							
r	*n*	$E(X_{r:n})$	$\mathrm{Var}(X_{r:n})$	*r*	*n*	$E(X_{r:n})$	$\mathrm{Var}(X_{r:n})$
1	5	0.549598	0.289870	6	8	2.998304	1.008345
2	5	1.215261	0.661445	7	8	3.824760	1.066195
3	5	2.032530	1.083064	8	8	4.810789	0.841091
4	5	3.050839	1.431655	1	9	0.307190	0.093782
5	5	4.339984	1.361012	2	9	0.651273	0.207489
1	6	0.459410	0.205728	3	9	1.038835	0.342187
2	6	1.000542	0.466561	4	9	1.478013	0.496439
3	6	1.644700	0.774588	5	9	1.978936	0.663636
4	6	2.420359	1.090715	6	9	2.554324	0.827093
5	6	3.366079	1.303996	7	9	3.220295	0.951131
6	6	4.534765	1.144777	8	9	3.997464	0.964849
1	7	0.394419	0.153073	9	9	4.912455	0.732598
2	7	0.849355	0.344257	1	10	0.276542	0.076208
3	7	1.378509	0.572319	2	10	0.583023	0.167409
4	7	1.999620	0.823835	3	10	0.924272	0.274646
5	7	2.735914	1.058535	4	10	1.306148	0.397699
6	7	3.618146	1.179800	5	10	1.735809	0.533784
7	7	4.687535	0.975570	6	10	2.222062	0.675267
1	8	0.345422	0.118109	7	10	2.775831	0.805647
2	8	0.737398	0.263383	8	10	3.410779	0.892534
3	8	1.185228	0.436466	9	10	4.144135	0.875366
4	8	1.700645	0.632706	10	10	4.997824	0.643857
5	8	2.298596	0.836192				

$$\times\left[\frac{1-F(y)}{1-F(x)}\right]^{\gamma_i}\frac{f(y)}{1-F(y)}\,dydx$$

$$=\frac{(1+\gamma)\theta c_{s-1}}{j+1}\int_0^{\theta}x^i\left\{\sum_{i=1}^{r}a_i(r)[1-F(x)]^{\gamma_i}\right\}$$

$$\times\frac{f(x)}{1-F(x)}\int_x^{\theta}y^{j+1}\sum_{i=r+1}^{s}a_i^{(r)}(s)\gamma_i$$

$$\times\left[\frac{1-F(y)}{1-F(x)}\right]^{\gamma_i}\frac{f(y)}{1-F(y)}\,dydx.$$

Proceeding in the same way as done in Theorem 3.1, we get the required result given in Theorem 3.3.

Remark 3.3 : Putting $m_i = m_j = m$ in (6) and using (8) and (9), the recurrence relation obtained in Theorem 3.3 reduces to the recurrence relation for product moments of generalized order statistics from Schabe distribution for Case 1, *i.e.*, when $m_1=m_2=...=m_{n-1}=m$.

Table 3 : Coefficients of the BLUE of location parameter.

θ, γ	*n*	*c*	$a_i, i = 1, 2, ..., (n\text{-}c)$				
5, 0.4	5	0	1.174002	-0.045998	-0.030956	-0.020297	-0.076751
		1	1.243120	-0.062883	-0.042369	-0.137868	
	6	0	1.146078	-0.034985	-0.025227	-0.017734	-0.012198
			-0.055934				
		1	1.191370	-0.045463	-0.032787	-0.023052	-0.090068
		2	1.260117	-0.060070	-0.043407	-0.156641	
	7	0	1.126204	-0.027337	-0.020764	-0.015423	-0.011214
			-0.008003	-0.043463			
		1	1.158981	-0.034448	-0.026158	-0.019423	-0.014115
			-0.064838				
		2	1.202724	-0.043188	-0.032827	-0.024406	-0.102302
	8	0	1.111166	-0.021845	-0.017261	-0.013385	-0.010183
			-0.007606	-0.005591	-0.035295		
		1	1.136413	-0.026961	-0.021296	-0.016505	-0.012548
			-0.009363	-0.049740			
		2	1.167286	-0.032747	-0.025879	-0.020070	-0.015271
			-0.073318				
		3	1.210339	-0.040277	-0.031864	-0.024745	-0.113453
	9	0	1.099326	-0.017793	-0.014495	-0.011626	-0.009173
			-0.007118	-0.005436	-0.004094	-0.029591	
		1	1.119614	-0.021631	-0.017616	-0.014121	-0.011133
			-0.008631	-0.006583	-0.039899		
		2	1.142906	-0.025726	-0.020955	-0.016803	-0.013253
			-0.010279	-0.055890			
		3	1.172770	-0.030633	-0.024968	-0.020037	-0.015821
			-0.081310				
	10	0	1.089737	-0.014732	-0.012295	-0.010128	-0.008228
			-0.006588	-0.005199	-0.004046	-0.003109	-0.025412
		1	1.106544	-0.017705	-0.014770	-0.012160	-0.009872
			-0.007898	-0.006226	-0.004837	-0.033075	
		2	1.124946	-0.020743	-0.017306	-0.014250	-0.011571
			-0.009259	-0.007300	-0.044517		
		3	1.147138	-0.024175	-0.020178	-0.016624	-0.013507
			-0.010817	-0.061837			
		4	1.176534	-0.028471	-0.023778	-0.019605	-0.015945
			-0.088734				

Table 4 : Coefficients of the BLUE of scale parameter.

θ, γ	*n*	*c*	$b_i, i = 1, 2, ..., (n\text{-}c)$				
5, 0.4	5	0	-0.440409	0.064656	0.043154	0.027975	0.304624
		1	-0.714739	0.131669	0.088455	0.494615	
	6	0	-0.423428	0.056332	0.040282	0.027997	0.018961
			0.279855				
		1	-0.650036	0.108755	0.078107	0.054605	0.408569
		2	-0.961890	0.175019	0.126281	0.660591	
	7	0	-0.411151	0.049461	0.037280	0.027407	0.019653

Table 4 continued...

Table 4 continued...

			0.013767	0.263583			
		1	-0.609929	0.092583	0.069995	0.051664	0.037245
			0.358442				
		2	-0.851753	0.140901	0.106864	0.079215	0.524774
	8	0	-0.401375	0.043708	0.034302	0.026361	0.019819
			0.014573	0.010490	0.252122		
		1	-0.581719	0.080257	0.063124	0.048649	0.036711
			0.027124	0.325854			
		2	-0.783976	0.118163	0.093151	0.072005	0.054551
			0.446106				
		3	-1.045930	0.163979	0.129562	0.100449	0.651940
	9	0	-0.393105	0.038859	0.031470	0.025046	0.019564
			0.014985	0.011249	0.008282	0.243649	
		1	-0.560149	0.070467	0.057161	0.045588	0.035705
			0.027442	0.020693	0.303094		
		2	-0.737086	0.101569	0.082529	0.065965	0.051809
			0.039964	0.395250			
		3	-0.948279	0.136272	0.110911	0.088839	0.069969
			0.542288				
	10	0	-0.385839	0.034748	0.028848	0.023608	0.019018
			0.015064	0.011721	0.008956	0.006720	0.237156
		1	-0.542685	0.062491	0.051946	0.042576	0.034366
			0.027289	0.021302	0.016343	0.286371	
		2	-0.702018	0.088791	0.073904	0.060671	0.049074
			0.039071	0.030603	0.359906		
		3	-0.881430	0.116542	0.097124	0.079861	0.064725
			0.051666	0.471513			
		4	-1.105578	0.149299	0.124576	0.102593	0.083314
			0.645796				

Table 5 : Coefficients of the BLUE of location parameter.

θ, γ	***n***	***c***	***a**$_i$*, ***i* = 1, 2, 3, ..., (*n* – *c*)**				
5, 0.85	5	0	1.193203	-0.032100	-0.024970	-0.019233	-0.116900
		1	1.265332	-0.043404	-0.033772	-0.188156	
	6	0	1.158835	-0.023945	-0.019377	-0.015533	-0.012346
			-0.087634				
		1	1.204093	-0.030611	-0.024769	-0.019852	-0.128862
		2	1.276799	-0.040715	-0.032966	-0.203118	
	7	0	1.135209	-0.018514	-0.015441	-0.012770	-0.010476
			-0.008531	-0.069476			
		1	1.166664	-0.022856	-0.019057	-0.015755	-0.012920
			-0.096077				
		2	1.211706	-0.028722	-0.023956	-0.019811	-0.139217
	8	0	1.117846	-0.014714	-0.012563	-0.010648	-0.008960
			-0.007487	-0.006215	-0.057260		
		1	1.141195	-0.017733	-0.015136	-0.012824	-0.010786
			-0.009007	-0.075709			

Table 5 continued...

Table 5 continued...

		2	1.172162	-0.021517	-0.018367	-0.015563	-0.013091
			-0.103624				
		3	1.217198	-0.026761	-0.022852	-0.019371	-0.148215
	9	0	1.104494	-0.011955	-0.010398	-0.008987	-0.007719
			-0.006588	-0.005588	-0.004712	-0.048547	
		1	1.122635	-0.014155	-0.012308	-0.010634	-0.009129
			-0.007787	-0.006601	-0.062021		
		2	1.145414	-0.016771	-0.014582	-0.012598	-0.010815
			-0.009225	-0.081422			
		3	1.176137	-0.020134	-0.017510	-0.015131	-0.012993
			-0.110370				
	10	0	1.093882	-0.009893	-0.008733	-0.007669	-0.006697
			-0.005815	-0.005022	-0.004314	-0.003686	-0.042053
		1	1.108456	-0.011554	-0.010197	-0.008950	-0.007813
			-0.006781	-0.005853	-0.005024	-0.052283	
		2	1.126023	-0.013454	-0.011873	-0.010421	-0.009096
			-0.007894	-0.006813	-0.066473		
		3	1.148469	-0.015770	-0.013918	-0.012218	-0.010665
			-0.009258	-0.086641			
		4	1.179102	-0.018804	-0.016600	-0.014576	-0.012728
			-0.116394				

Table 6 : Coefficients of the BLUE of scale parameter.

θ, γ	***n***	***c***	b_i, $i = 1, 2, 3, \ldots, (n - c)$				
5, 0.85	5	0	-0.355696	0.026094	0.020231	0.015519	0.293852
		1	-0.537009	0.054509	0.042358	0.440142	
	6	0	-0.334834	0.021046	0.016969	0.013542	0.010706
			0.272571				
		1	-0.475602	0.041780	0.033738	0.026975	0.373109
		2	-0.686115	0.071035	0.057472	0.557607	
	7	0	-0.320084	0.017415	0.014472	0.011917	0.009725
			0.007869	0.258686			
		1	-0.437204	0.033581	0.027936	0.023030	0.018823
			0.333834				
		2	-0.593710	0.053967	0.044956	0.037123	0.457664
	8	0	-0.308904	0.014685	0.012495	0.010547	0.008832
			0.007337	0.006049	0.248960		
		1	-0.410421	0.027812	0.023682	0.020007	0.016770
			0.013949	0.308201			
		2	-0.536484	0.043217	0.036836	0.031157	0.026154
			0.399119				
		3	-0.709943	0.063413	0.054108	0.045825	0.546598
	9	0	-0.300024	0.012568	0.010896	0.009382	0.008022
			0.006811	0.005742	0.004807	0.241797	
		1	-0.390377	0.023525	0.020407	0.017582	0.015045
			0.012784	0.010788	0.290245		
		2	-0.496977	0.035768	0.031051	0.026777	0.022937

Table 6 continued...

Table 6 continued...

			0.019514	0.360931			
		3	-0.633168	0.050675	0.044027	0.038003	0.032590
			0.467874				
	10	0	-0.292730	0.010887	0.009583	0.008386	0.007294
			0.006304	0.005415	0.004622	0.003920	0.236320
		1	-0.374630	0.020220	0.017805	0.015588	0.013566
			0.011733	0.010085	0.008614	0.277018	
		2	-0.467705	0.030287	0.026687	0.023379	0.020363
			0.017628	0.015169	0.334193		
		3	-0.580557	0.041932	0.036970	0.032412	0.028255
			0.024483	0.416505			
		4	-0.727816	0.056520	0.049864	0.043748	0.038170
			0.539514				

Table 7 : Coefficients of the BLUE of location parameter

θ, γ	n	c	$a_i, i = 1, 2, 3, ..., (n-c)$				
6, 0.85	5	0	1.193203	-0.032100	-0.024970	-0.019233	-0.116900
		1	1.265332	-0.043404	-0.033772	-0.188156	
	6	0	1.158835	-0.023945	-0.019377	-0.015533	-0.012346
			-0.087634				
		1	1.204093	-0.030611	-0.024769	-0.019852	-0.128862
		2	1.276798	-0.040715	-0.032966	-0.203118	
	7	0	1.135209	-0.018514	-0.015441	-0.012771	-0.010477
			-0.008531	-0.069476			
		1	1.166664	-0.022855	-0.019057	-0.015756	-0.012920
			-0.096077				
		2	1.211706	-0.028722	-0.023955	-0.019811	-0.139217
	8	0	1.117846	-0.014714	-0.012563	-0.010648	-0.008960
			-0.007487	-0.006215	-0.057260		
		1	1.141195	-0.017733	-0.015136	-0.012824	-0.010786
			-0.009007	-0.075709			
		2	1.172162	-0.021517	-0.018368	-0.015563	-0.013091
			-0.103624				
		3	1.217198	-0.026760	-0.022852	-0.019371	-0.148215
	9	0	1.089113	-0.011798	-0.010262	-0.008871	-0.007620
			-0.006505	-0.005519	-0.023741	-0.014798	
		1	1.122635	-0.014155	-0.012308	-0.010634	-0.009129
			-0.007787	-0.006601	-0.062021		
		2	1.145414	-0.016771	-0.014582	-0.012599	-0.010815
			-0.009225	-0.081422			
		3	1.176137	-0.020134	-0.017509	-0.015131	-0.012993
			-0.110370				
	10	0	1.093882	-0.009893	-0.008734	-0.007669	-0.006697
			-0.005816	-0.005022	-0.004314	-0.003686	-0.042053
		1	1.108456	-0.011553	-0.010197	-0.008950	-0.007813
			-0.006782	-0.005853	-0.005024	-0.052283	
		2	1.126022	-0.013454	-0.011873	-0.010421	-0.009096
			-0.007894	-0.006813	-0.066473		
		3	1.148469	-0.015770	-0.013918	-0.012217	-0.010665
			-0.009258	-0.086641			
		4	1.179102	-0.018804	-0.016600	-0.014576	-0.012728
			-0.116394				

Table 8 : Coefficients of the BLUE of scale parameter.

θ, γ	***n***	***c***	b_i, i = 1, 2, 3, ..., ($n - c$)				
6, 0.85	5	0	-0.296414	0.021745	0.016859	0.012933	0.244876
		1	-0.447507	0.045424	0.035298	0.366785	
	6	0	-0.279028	0.017538	0.014141	0.011285	0.008922
			0.227143				
		1	-0.396335	0.034817	0.028115	0.022479	0.310925
		2	-0.571762	0.059196	0.047894	0.464672	
	7	0	-0.266736	0.014513	0.012060	0.009930	0.008104
			0.006558	0.215572			
		1	-0.364337	0.027984	0.023279	0.019192	0.015686
			0.278195				
		2	-0.494758	0.044973	0.037463	0.030936	0.381387
	8	0	-0.257420	0.012237	0.010413	0.008789	0.007360
			0.006114	0.005041	0.207467		
		1	-0.342017	0.023176	0.019735	0.016672	0.013976
			0.011624	0.256834			
		2	-0.447070	0.036014	0.030697	0.025964	0.021796
			0.332599				
		3	-0.591619	0.052843	0.045091	0.038187	0.455498
	9	0	-0.273596	0.010569	0.009164	0.007892	0.006749
			0.005731	0.004833	0.008842	0.190255	
		1	-0.325314	0.019604	0.017006	0.014652	0.012538
			0.010654	0.008990	0.241871		
		2	-0.414147	0.029806	0.025876	0.022314	0.019114
			0.016262	0.300776			
		3	-0.527640	0.042228	0.036689	0.031670	0.027158
			0.389895				
	10	0	-0.243942	0.009072	0.007986	0.006988	0.006078
			0.005254	0.004512	0.003851	0.003266	0.196933
		1	-0.312191	0.016850	0.014838	0.012990	0.011305
			0.009778	0.008404	0.007179	0.230848	
		2	-0.389755	0.025240	0.022239	0.019483	0.016969
			0.014690	0.012640	0.278494		
		3	-0.483798	0.034943	0.030808	0.027010	0.023545
			0.020404	0.347087			
		4	-0.606513	0.047100	0.041553	0.036457	0.031808
			0.449596				

Table 9 : Variances and covariance of the BLUEs when μ = 0 and σ = 1.

θ, γ	***n***	***c***	**Var(μ*)**	**Var(σ*)**	**Cov(μ*, σ*)**
5, 0.4	5	0	0.122568	0.137932	-0.052783
		1	0.131794	0.283275	-0.089403
	6	0	0.080178	0.105157	-0.033592
		1	0.084171	0.205120	-0.053572
		2	0.090107	0.327274	-0.080500
	7	0	0.056314	0.083747	-0.023043
		1	0.058348	0.158545	-0.035376
		2	0.061013	0.239990	-0.050109
	8	0	0.041632	0.068762	-0.016677
		1	0.042788	0.127765	-0.024937
		2	0.044180	0.187493	-0.034054
		3	0.046095	0.258402	-0.045708
	9	0	0.031987	0.057751	-0.012566
		1	0.032699	0.106003	-0.018427
		2	0.033505	0.152509	-0.024549
		3	0.034526	0.203578	-0.031770
	10	0	0.025325	0.049363	-0.009770
		1	0.025790	0.089868	-0.014111
		2	0.026293	0.127588	-0.018467

Table 9 continued...

Table 9 continued...

		3	0.026894	0.166828	-0.023321
		4	0.027682	0.212664	-0.029332
5, 0.85	5	0	0.2536267	0.0966207	-0.0799427
		1	0.2714023	0.2089393	-0.1246253
	6	0	0.1719426	0.0708269	-0.0521935
		1	0.179673	0.1456136	-0.076238
		2	0.1919798	0.2487872	-0.1118713
	7	0	0.1240149	0.0545917	-0.0365276
		1	0.1279397	0.1090026	-0.051141
		2	0.1335134	0.1762961	-0.0705079
	8	0	0.093565	0.0435919	-0.0268777
		1	0.0957797	0.0854586	-0.036507
		2	0.0986954	0.1337759	-0.0483761
		3	0.10291	0.1962989	-0.0646091
	9	0	0.0730482	0.035738	-0.0205411
		1	0.0743979	0.0692186	-0.0272632
		2	0.0760814	0.1060874	-0.0351416
		3	0.0783393	0.1504569	-0.0451508
	10	0	0.0585835	0.0299051	-0.0161712
		1	0.0594555	0.0574439	-0.0210717
		2	0.0605004	0.0867772	-0.026608
		3	0.0618288	0.1203526	-0.0332863
		4	0.0636339	0.1620681	-0.0419639

Table 10 : Variances and covariance of the BLUEs when $\mu = 0$ and $\sigma = 1$.

θ, γ	***n***	***c***	**Var(μ*)**	**Var(σ*)**	**Cov(μ*, σ*)**
6, 0.85	5	0	0.365222	0.096621	-0.095931
		1	0.390819	0.208939	-0.149550
	6	0	0.247597	0.070827	-0.062632
		1	0.258729	0.145614	-0.091486
		2	0.276451	0.248787	-0.134246
	7	0	0.178581	0.054592	-0.043833
		1	0.184233	0.109003	-0.061369
		2	0.192259	0.176296	-0.084609
	8	0	0.134734	0.043592	-0.032253
		1	0.137923	0.085459	-0.043808
		2	0.142121	0.133776	-0.058051
		3	0.148190	0.196299	-0.077531
	9	0	0.103725	0.035634	-0.024301
		1	0.107133	0.069219	-0.032716
		2	0.109557	0.106087	-0.042170
		3	0.112809	0.150457	-0.054181
	10	0	0.084360	0.029905	-0.019405
		1	0.085616	0.057444	-0.025286
		2	0.087121	0.086777	-0.031930
		3	0.089033	0.120353	-0.039944
		4	0.091633	0.162068	-0.050357

Table 11 : BLUEs of μ and σ with Mean Square Errors.

Parameters	***n***	***c***	**μ***	**MSE(μ*)**	**σ***	**MSE(σ*)**
θ = 5, γ = 0.4	6	0	0.000639	0.078787	1.001020	0.104566
	6	2	-0.000913	0.088567	1.001876	0.330248
	8	0	-0.005067	0.037978	0.997523	0.069660
	8	3	-0.005444	0.042623	0.999974	0.260623
	10	0	0.002964	0.026718	0.999706	0.049320
	10	3	0.002556	0.028357	1.003143	0.167764
θ = 5, γ = 0.85	6	0	0.000421	0.172920	0.997054	0.070857
	6	2	0.000937	0.193057	0.995147	0.244950
	8	0	0.002259	0.095968	0.997062	0.043559
	8	3	0.002722	0.104714	0.995083	0.194713
	10	0	-0.002353	0.057220	1.001227	0.030602
	10	3	-0.002547	0.060477	1.002259	0.120544
θ = 6, γ = 0.85	6	0	0.005269	0.236028	0.999311	0.070598
	6	2	-0.005732	0.262637	1.000518	0.245854
	8	0	0.001136	0.134333	0.999140	0.043918
	8	3	0.002429	0.147329	0.994840	0.195167
	10	0	0.000441	0.083616	1.001250	0.030022
	10	3	0.001581	0.088518	0.996131	0.120952

4. BLUEs of μ and σ

Let $X_{1:n} \le X_{2:n} \le \ldots \le X_{n-c:n}$, c = 0, 1, ..., n – 1, denote type-II right censored sample from the location-scale parameter Schabe distribution in (13). Let us denote $Z_{r:n} = (X_{r:n} - \mu)/\sigma$, $E(Z_{r:n}) = \mu_{r:n}$, $1 \le r \le (n-c)$ and $Cov(Z_{r:n}, Z_{s:n}) = \sigma_{r,s:n} = \mu_{r,s:n} - \mu_{r:n}\mu_{s:n}$, $1 \le r < s \le (n-c)$. We shall use the following notations :

$$X = \left(X_{1:n}, X_{2:n}, \ldots, X_{n-c:n}\right)^T,$$

$$\mu = \left(\mu_{1:n}, \mu_{2:n}, \ldots, \mu_{n-c:n}\right)^T,$$

$$1 = (1, 1, \ldots, 1)^T$$

and $\sum = (\sigma_{r,s:n})$, $1 \le r, s \le n-c$.

The BLUEs of μ and σ are given by

$$\mu^* = \left\{\frac{\mu^T\Sigma^{-1}\mu 1^T\Sigma^{-1} - \mu^T\Sigma^{-1}1\mu^T\Sigma^{-1}}{\left(\mu^T\Sigma^{-1}\mu\right)\left(1^T\Sigma^{-1}1\right) - \left(\mu^T\Sigma^{-1}1\right)^2}\right\}X$$

$$= \sum_{r=1}^{n-c} a_r X_{r:n} \quad (18)$$

and

$$\sigma^* = \left\{\frac{1^T\Sigma^{-1}1\mu^T\Sigma^{-1} - 1^T\Sigma^{-1}\mu 1^T\Sigma^{-1}}{\left(\mu^T\Sigma^{-1}\mu\right)\left(1^T\Sigma^{-1}1\right) - \left(\mu^T\Sigma^{-1}1\right)^2}\right\}X$$

$$= \sum_{r=1}^{n-c} b_r X_{r:n}, \quad (19)$$

and the variances and covariance of these BLUEs are given by

$$Var\left(\mu^*\right) = \sigma^2\left\{\frac{\mu^T\Sigma^{-1}\mu}{\left(\mu^T\Sigma^{-1}\mu\right)\left(1^T\Sigma^{-1}1\right) - \left(\mu^T\Sigma^{-1}1\right)^2}\right\} = \sigma^2 V_1; \quad (20)$$

$$Var\left(\sigma^*\right) = \sigma^2\left\{\frac{1^T\Sigma^{-1}1}{\left(\mu^T\Sigma^{-1}\mu\right)\left(1^T\Sigma^{-1}1\right) - \left(\mu^T\Sigma^{-1}1\right)^2}\right\} = \sigma^2 V_2 \quad (21)$$

and

$$Cov\left(\mu^*, \sigma^*\right) = \sigma^2\left\{\frac{-\mu^T\Sigma^{-1}1}{\left(\mu^T\Sigma^{-1}\mu\right)\left(1^T\Sigma^{-1}1\right) - \left(\mu^T\Sigma^{-1}1\right)^2}\right\}$$

$$= \sigma^2 V_3, \quad (22)$$

respectively [Arnold *et al.* (1992)].

Tables 3 - 8 display the coefficients of the BLUEs of location and scale parameters for type-II right censored samples of sizes n=5(1)10 for three sets of particular values of parameters θ and γ, *viz.* (θ = 5, γ = 0.4), (θ = 5, γ = 0.85) and (θ = 6, γ = 0.85) and different censoring cases c = 0(1)([n/2] – 1). The coefficients of the BLUEs in Tables 3 - 8 are checked by using the conditions :

$$\sum_{r=1}^{n-c} a_r = 1$$

and

$$\sum_{r=1}^{n-c} b_r = 0.$$

The corresponding variances and covariances of the BLUEs are presented in Tables 9 - 10, for the above mentioned three sets of particular values of θ and γ. We see that the variances of the BLUEs increase as the censoring level increases while the variances of the BLUEs decrease as the sample size increases. In addition, we see that the covariance of the BLUEs decreases as the censoring level increases while the covariance of the BLUEs increases as the sample size increases.

5. Simulation Study

In this section, we perform simulation study to obtain BLUEs of location and scale parameters of the distribution. We generate 10000 random samples of sizes n=6, 8 and 10 for different sets of particular values of parameters. Then, we compute the BLUEs of the location and scale parameters with different censoring levels. The coefficients needed to obtain the BLUEs can be seen in Tables 3-8. The estimates and their mean square errors are shown in Table 11. We observe from Table 11 that BLUEs are closer to true parameter values and the mean square errors of both the estimates μ^* and s^* decrease as sample size increases.

Acknowledgement

The authors are grateful to the referee and the Chief-Editor for their valuable comments which led to an improvement in the presentation of the paper.

References

Ahmad, A. A. (2007). Relations for single and product moments of generalized order statistics from doubly truncated Burr type XII distribution. *Journal of the Egyptian Mathematical Society*, **15**, 117-128.

Ahmad, A. A. and M. Fawzy (2003). Recurrence relations for single moments of generalized order statistics from doubly truncated distributions. *Journal of Statistical Planning and Inference*, **117**, 241-249.

Ahsanullah, M. (2000). Generalized order statistics from exponential distribution. *Journal of Statistical Planning and Inference*, **85**, 85-91.

Arnold, B. C., N. Balakrishnan and H. N. Nagaraja (1992). *A First Course in Order Statistics*. John Wiley, New York.

Athar, H. and H. M. Islam (2004). Recurrence relations between single and product moments of generalized order statistics from a general class of distribution. *Metron.*, **LXII**, 327-337.

Balakrishnan, N. and M. P. Chandramouleeswaran (1996). BLUEs of location and scale parameters of Laplace distribution based on Type-II censored samples and associated inference. *Microelectron Reliab.*, **36**, 371-374.

Cramer, E. and U. Kamps (2000). Relations for expectations of functions of generalized order statistics. *Journal of Statistical Planning and Inference*, **89**, 79-89.

Jabeen, R., A. Ahmad, N. Feroze and G. M. Gilani (2013). Estimation of location and scale parameters of Weibull distribution using generalized order statistics under Type II singly and doubly censored data. *Int J Adv Sci Technol.*, **55**, 67-80.

Kamps, U. (1995a). A concept of generalized order statistics. B.G. Teubner Stuttgart.

Kamps, U. (1995 b). A concept of generalized order statistics.

Journal of Statistical Planning and Inference, **48**, 1-23.

Kamps, U. and E. Cramer (2001). On distribution of generalized order statistics. *Statistics*, **35**, 269-280.

Kamps, U. and U. Gather (1997). Characteristic property of generalized order statistics for exponential distribution. *Appl. Math. (Warsaw)*, **24**, 383-391.

Keseling, C. (1999). Conditional distributions of generalized order statistics and some characterizations. *Metrika*, **49**, 27-40.

Khan, R. U., Z. Anwar and H. Athar (2007). Recurrence relations for single and product moments of generalized order statistics from doubly truncated Weibull distribution. *Aligarh Journal of Statistics*, **27**, 69-79.

Khan, R. U., D. Kumar and H. Athar (2010). Moments of generalized order statistics from Erlang-truncated exponential distribution and its characterization. *International Journal of Statistics and System*, **5**, 455-464.

Mahmoud, M. A. W., K. S. Sultan and S. M. Amer (2003). Order statistics from inverse weibull distribution and associated inference. *Comput. Stat. Data. Anal.*, **42**, 149-163.

Pawlas, P. and D. Szynal (2001). Recurrence Relations for single and product moments of generalized order statistics from Pareto, generalized Pareto and Burr distributions. *Commun. Statist.-Theory Meth.*, **30(4)**, 739-746.

Samuel, P. and P. Y. Thomas (1997). Estimation of location and scale parameters of U-shaped distribution. *J. Int. Soc. Agric. Stat.*, **50**, 75-94.

Saran, J. and A. Pandey (2004). Recurrence relations for single and product moments of generalized order statistics from linear exponential distribution. *Journal of Applied Statistical Science*, **13(4)**, 323-333.

Saran, J. and A. Pandey (2009). Recurrence relations for single and product moments of generalized order statistics from linear exponential and Burr distributions. *Journal of Statistical Theory and Applications*, **8**, 383-391.

Schabe, H. (1994). Constructing lifetime distributions with bathtub shaped failure rate from DFR distributions. *Microelectron Reliab.*, **34**, 1501-1508.