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ORIGINAL ARTICLE

STUDY OF A POWER FUNCTION AS A TWO PARAMETRIC BASIC PROBABILITY DISTRIBUTION

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Abstract : A power function having two constants is being studied here as a basic probability distribution with two parameters, some mathematical and statistical properties of this new distribution have been studied. Its utility for failure data has been shown by fitting a failure data on this distribution.

Key words : Parametric distribution, Failure data, Power function, Reliability function

1. Introduction

A good number of research papers are available on developing new probability distributions and establishing their utility in different fields of applications specially in life testing and reliability analysis. Mukherjee and Islam (1983) and Siddiqui *et al*. (1992, 1994, 1995, 2016a, 2016b) introduced new finite infinite range probability distributions and established their utilities in life testing analysis.

2. The Probability Density Function of the Distribution and its Characteristics

In this section a finite range basic distribution has been characterized through a power function with two parameters α and θ.

$$f(x) = \frac{\alpha-\theta}{\alpha} x^{-\theta/\alpha}, \; 0 < x < 1, \; \theta < \alpha, \; \alpha > 0 \qquad (1)$$

This form of the function is taken as its probability density function.

2.1 Cumulative Distribution Function

The cumulative distribution function for the p.d.f. in (1) can be obtained as

$$F(x) = \int_0^x f(x)dx$$

$$= \int_0^x \frac{\alpha-\theta}{\alpha} x^{-\theta/\alpha} dx$$

$$= \frac{(\alpha-\theta)}{\alpha}\left[\frac{x^{1-\theta/\alpha}}{1-\theta/\alpha}\right]_0^x$$

$$F(x) = x^{1-\theta/\alpha} \qquad (2)$$

2.2 Reliability Function

The reliability function for the p.d.f. in (1) can be obtained as

$$R(x) = 1 - F(x)$$

$$R(x) = 1 - x^{1-\theta/\alpha} \qquad (3)$$

2.3 Hazard Rate Function

The hazard rate function for the p.d.f. in (1) can be obtained as

$$h(x) = \frac{f(x)}{R(x)}$$

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$$h(x)=\frac{\frac{\alpha-\theta}{\alpha}x^{-\theta/\alpha}}{1-x^{-\theta/\alpha}} \tag{4}$$

The probability density function, cumulative distribution function, reliability function and hazard rate function of the distribution have been plotted in Graphs 1 - 4 respectively.

2.4 Moment Generating Function (m.g.f.)

The moment generating function (m.g.f.) of the distribution in (1) can be obtained as follows:

$M_x(t) = E(e^{tx})$

$$=\int_0^1 e^{tx} f(x)dx$$

$$=\int_0^1 e^{tx}\frac{\alpha-\theta}{\alpha}x^{-\frac{\theta}{\alpha}}dx$$

$$=\frac{(\alpha-\theta)}{\alpha}\int_0^1 x^{-\theta/\alpha}\left[1+\frac{tx}{1!}+\frac{(tx)^2}{2!}+---+\frac{(tx)^n}{n!}\right]dx$$

$$M_x(t)=(\alpha-\theta)\sum_{t=0}^{n}\frac{t^r}{r!(r+1)\alpha-\theta} \tag{5}$$

2.5 Characteristic Function

The characteristic function (c.f.) of the distribution can be obtained as

$\phi_x(t) = E(e^{itx})$, using same steps of m.g.f., finally it comes as

$$\phi_x(t)=(\alpha-\theta)\sum_{r=0}^{n}\frac{it^r}{r!(r+1)\alpha-\theta} \tag{6}$$

2.6 *r*th Moment about Origin

The *r*th moment about origin is given by

$$\mu_r'=E\left(X^r\right)$$

$$=\int_0^1\frac{\alpha-\theta}{\alpha}x^{-\frac{\theta}{\alpha}}x^r dx$$

$$\mu_r'=\frac{(\alpha-\theta)}{(r+1)\alpha-\theta} \tag{7}$$

This in turn gives the following results directly

2.7 Mean

$$\text{Mean} = E(X) = \mu_1' = \frac{\alpha-\theta}{2\alpha-\theta} \tag{8}$$

2.8 Variance

$$V(X)=\frac{\alpha-\theta}{3\alpha-\theta}-\left(\frac{\alpha-\theta}{2\alpha-\theta}\right)^2=\frac{\alpha(\alpha-\theta)}{(3\alpha-\theta)(2\alpha-\theta)} \tag{9}$$

2.9 Median

To obtain the median we proceed as follows

$$\int_0^{Me}\frac{\alpha-\theta}{\alpha}x^{-\theta/\alpha}dx=\frac{1}{2}$$

$$Median=e^{\frac{-3.010}{1-\theta/\alpha}} \tag{10}$$

3. Single Moment of Order Statistics

To obtain single moment of order statistics we proceed as follows

We know that the pdf of $X_{r:n}$, $(1\le r\le n)$ is given by

$$f_{r:n}(x)=C_{r:n}[F(x)]^{r-1}[1-F(x)]^{n-r}f(x),\ -\infty\ x<\infty \tag{11}$$

where, $C_{r:n}=\frac{n!}{(r\text{ -}1):(n-r)!}$

Therefore, for $p\to n$ single moment of order statistics is given by

$$\alpha_{r:n}^{p}=C_{r:n}\int_0^1 x^p[F(x)]^{r-1}[1-F(x)]^{n-r}f(x)dx$$

In view of Equations (1), (2) and (3), we have

$$\alpha_{r:n}^{p}=C_{r:n}\int_0^1 x^p(x^{1-\theta/\alpha})^{r-1}(1-x^{1-\theta/\alpha})^{n-r}\frac{\alpha-\theta}{\alpha}x^{-\theta/\alpha}dx$$

On putting $x^{1-\theta/\alpha}=t$ in above Equation, we get

$$\alpha_{r:n}^{p}=C_{r:n}\int_0^1 t^{\left(\frac{p\alpha+r\alpha-r\theta}{\alpha-\theta}\right)-1}(1-t)^{n-r}dt$$

$$\alpha_{r:n}^{p}=C_{r:n}\beta\left[\frac{p\alpha+r\alpha-r\theta}{\alpha-\theta},n-r+1\right]$$

3.1 Product moment of order statistics

To obtain product moment of order statistics, we proceed as follows

We know that the joint pdf of $X_{r:n}$ and $X_{s:n}$, $(1\le r\le s\le n)$ is given by

$$f_{r,s:n}(x)=C_{r,s:n}[F(x)]^{r-1}[F(y)-F(x)]^{s-r-1}$$

$$\times [1-F(y)]^{n-s} f(x) f(y)$$

$$-\infty < x < y < \infty \qquad (12)$$

where, $C_{r,s:n} = \dfrac{n!}{(r-1)!\,(s-n-r)!\,(n-s)!}$

Therefore, for p, $q \rightarrow n$, product moment of order statistics is given by

$$\alpha_{r,s:n}^{p,q} = C_{r,s:n} \int_0^1 \int_x^1 x^p y^b [F(x)]^{r-1} [F(y)-F(x)]^{s-r-1}$$

$$\times [1-F(y)]^{n-s} f(x) f(y) dxdy$$

Expanding the terms inside the brackets binomially

$$= C_{r,s:n}(-1)^{i+j} \sum_{i=0}^{s-r-1} \binom{s-r-1}{i} \sum_{j=0}^{n-s} \binom{n-s}{j}$$

$$\times \int_0^1 x^p [F(x)]^{r+i-1} f(x) dx \times I(x)$$

and,

$$I(x) = \int_x^1 y^p [F(y)]^{s-r-i+j-1} f(y) dy$$

From Equations (1) and (2), we set

$$I(x) = \int_x^1 y^q \left(y^{1-\theta/\alpha} \right)^{s-r-i+j-1} \frac{\alpha-\theta}{\alpha} y^{-\theta/\alpha} dy$$

After some algebraic computations

$$I(x) = \frac{\alpha-\theta}{q\alpha+(s-r-i+j)(\alpha-\theta)} \times (1-x)^{q+(s-r-i+j)(1-\theta/\alpha)}$$

Therefore,

$$\alpha_{r,s:n}^{p,q} = C_{r,s:n}(-1)^{i+j} \sum_{i=0}^{s-r-1} \binom{s-r-1}{i} \sum_{j=0}^{n-s} \binom{n-s}{j}$$

$$\times \frac{\alpha-\theta}{q\alpha+(s-r-i+j)(\alpha-\theta)} \times \frac{\alpha-\theta}{\alpha}$$

$$\times \int_0^1 x^p (1-x^{1-\theta/\alpha})^{r+i-1} x^{-\theta/\alpha} (1-x)^{q+(s-r-i+j)(1-\theta/\alpha)} dx \qquad (13)$$

Proceeding similarly as single moment of order statistics.

We will get product moment of order statistics

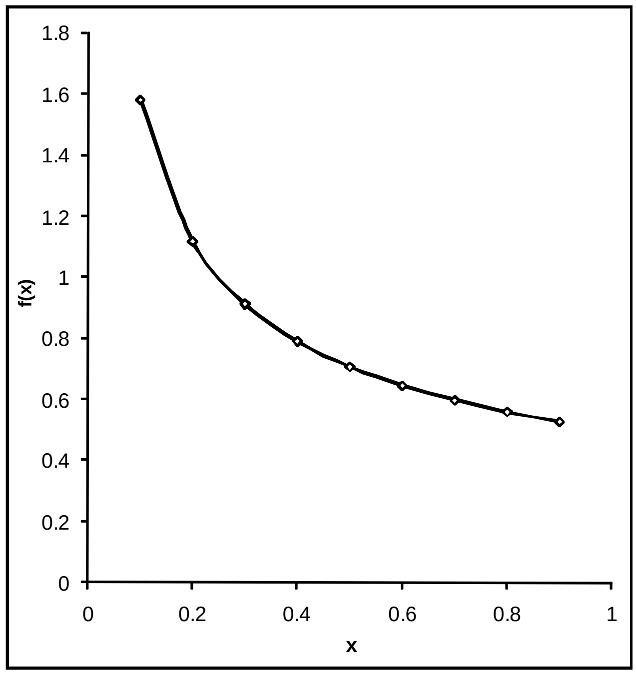
$$\alpha_{r,s:n}^{p,q} = C_{r,s:n}(-1)^{i+j} \sum_{i=0}^{s-r-1} \binom{s-r-1}{i} \sum_{j=0}^{n-s} \binom{n-s}{j}$$

$$\times \frac{(\alpha-\theta)^2}{\alpha(q\alpha+(s-r-i+j)(\alpha-\theta))}$$

$$\beta\left[p + \left(1-\frac{\theta}{\alpha}\right)(r+i), q+(s-r-i+j)\left(1-\theta/\alpha\right)+1 \right] \qquad (14)$$

Fitting of the distribution

We have considered the following data consisting of 100 observations on breaking stress of carbon fibers

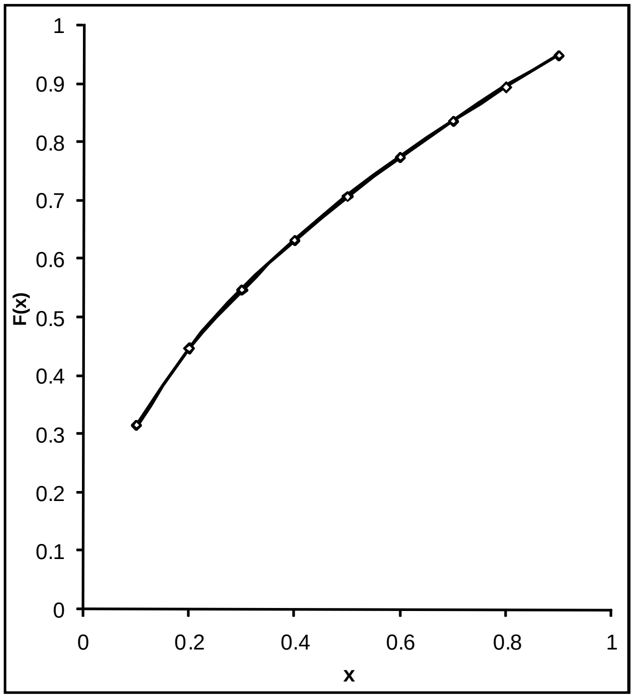


Graph 1 : Probability Density Function.

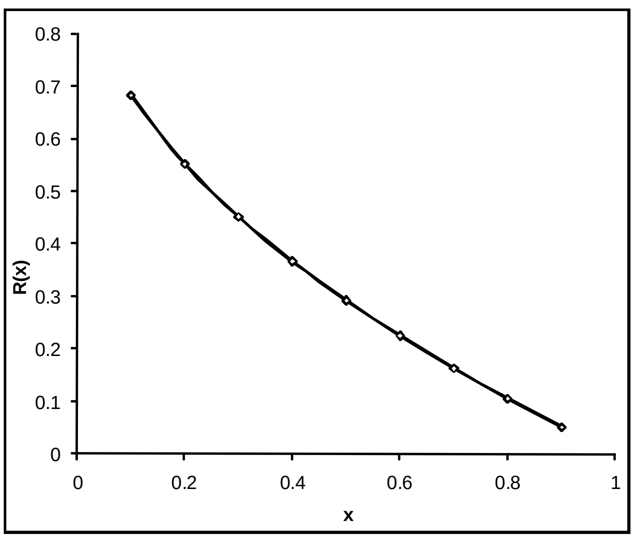
(in Gba). This data set is given by Nicholas and Padgett (2006).

3.70 2.74 2.73 2.50 3.60 3.11 3.27 2.87 1.47 3.11 4.42 2.41 3.19 3.22 1.69 3.28 3.09 1.87 3.15 4.90 3.75 2.43 2.95 2.97 3.39 2.96 2.53 2.67 2.93 3.22 3.39 2.81 4.20 3.33 2.55 3.31 3.31 2.85 2.56 3.56 3.15 2.35 2.55 2.59 2.38 2.81 2.77 2.17 2.83 1.92 1.41 3.68 2.97 1.36 0.98 2.76 4.91 3.68 1.84 1.59 3.19 1.57 0.81 5.56 1.73 1.59 2.00 1.22 1.12 1.71 2.17 1.17 5.08 2.48 1.18 3.51 2.17 1.69 1.25 4.38 1.84 0.39 3.68 2.48 0.85 1.61 2.79 4.70 2.03 1.80 1.57 1.08 2.03 1.61 2.12 1.89 2.88 2.82 2.05 3.65

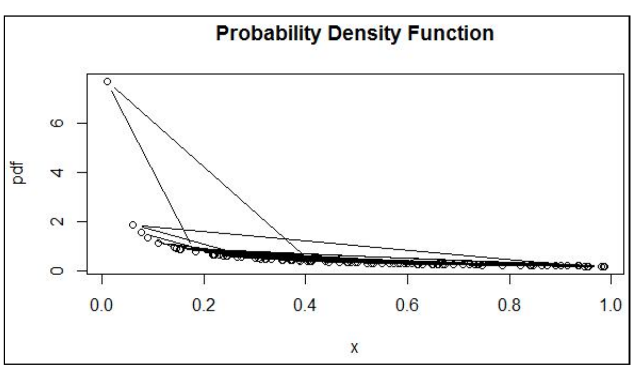
We have fitted the Cumulative Distribution Function, Probability Density Function, Reliability Function and Hazard Rate Function of the model for

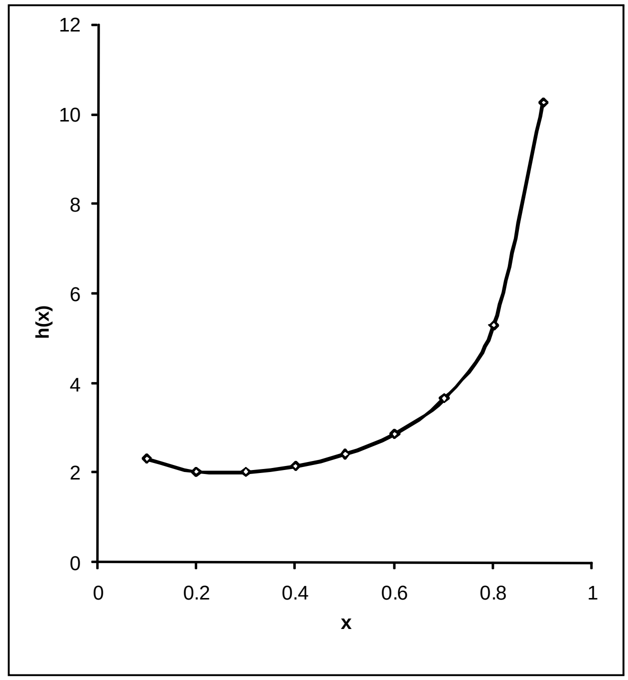


Graph 2 : Cumulative Density Function.

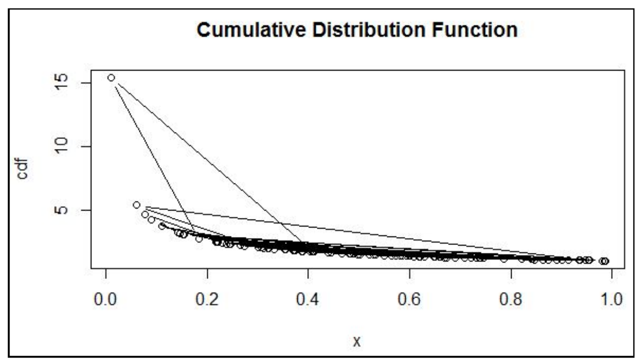


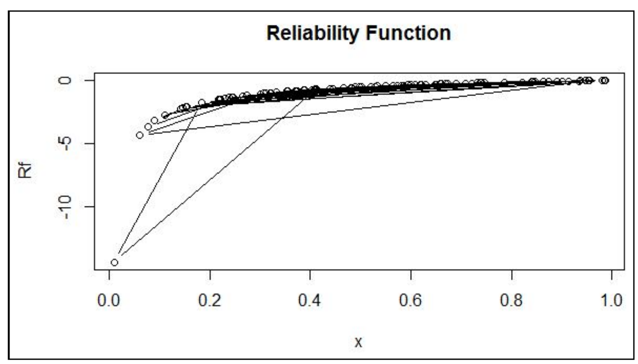
Graph 3 : Reliability function.

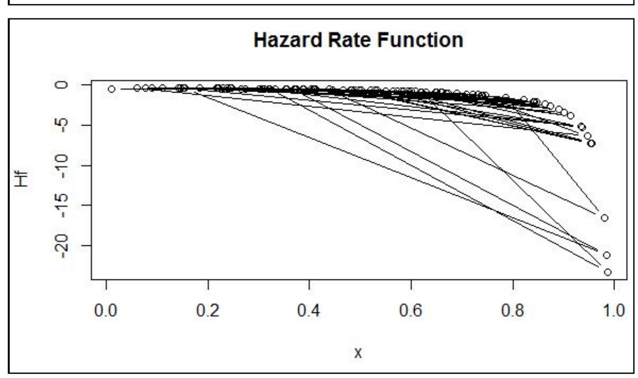


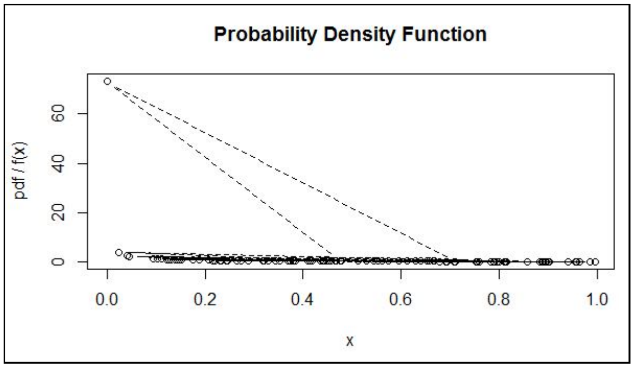


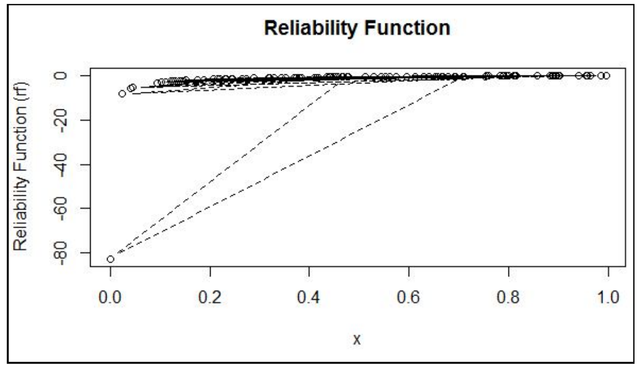
Graph 4 : Hazard rate function.

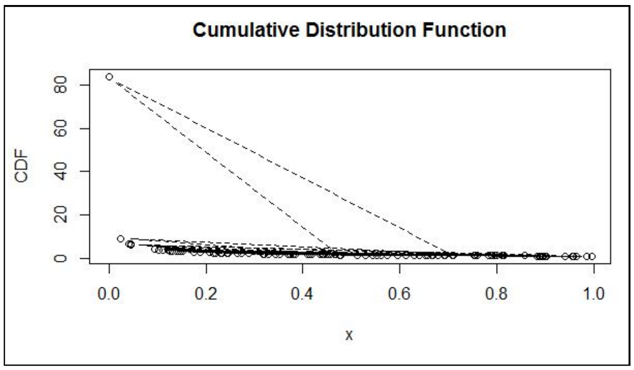


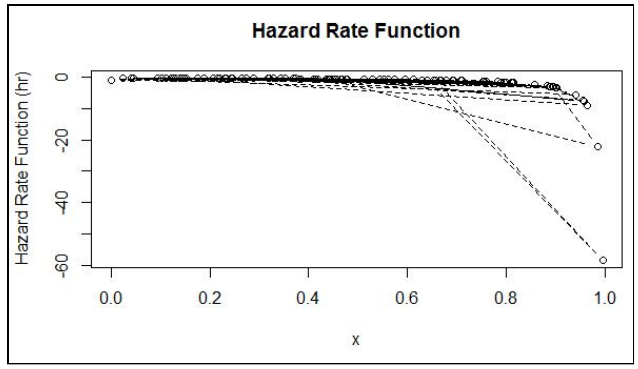












above set of data after converting data into percentage. So, that data will lie between limits (0 to 1) of the distribution for parameter value ($\alpha = 5$ & $\theta = 4$) using R software.

We can also fit this distribution to a random number generated from uniform distribution with the parameter value (α=5 & θ=4) using R software.

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