Geometer's Sketchpad and MSW logo in mathematics classroom instruction: a comparative analysis

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Abstract

Geometer Sketchpad and MSW Logo programs have been introduced in Bachelor of Science Education degree courses along with other software programs including a course on ICT in mathematics education at the University of Botswana. Sketchpad is known as a tool of dynamic geometry and Logo, basically as a programming language. Current trends of instruction involving technology tend to avoid programming language and some critics argue that the language was now given less prominence in enhancement of thinking skills. In this paper, both programs are used to illustrate how to integrate ICT into the teaching the topic on *Rotational Symmetry* from the Botswana General Certificate of Secondary Education (BGCSE). While the programs indicate that the topic can be taught in two ways that were not similar, it is illustrated that the two programs can be used to describe and investigate rotational geometry to achieve the same stipulated objectives with a difference. At the end of the paper, some differences in instruction using the two programs are identified as well as advantages and limitations.

Keywords : *Geometer's sketchpad, MSW logo, technology in mathematics instruction.*

Introduction

Educational studies have indicated that the use of some computer programs can be effective and versatile in instruction. These studies have shown that programs can contribute to innovative student-centered

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learning environment where teachers act as coaches, while remaining in firm control of the learning environment (Smeets and Mooij (2001)). Broekman *et al.* (2002) argued that programs promised power for students to control over their own learning, and promised to give voice to learners. Sibiya (2003), summarizing reviews and critiques of several studies on *Information and Communication Technology* (ICT), argued that ICT provided motivation and variety, generated enthusiasm, interest and involvement, maintained attention and enjoyment, enhanced thinking and problem-solving skills. Some educationists consider ICT as the only way to go if not a substitute for conventional teaching and learning resources (Broekman *et al.* (2002)). Its interactive testing and review mechanism, together with "a let-s-go-back and look-at-that-again-loop" is believed to offer the best of all worlds of learning (The Star Newspaper (2000)).

Some generic programs, i.e., general purpose programs such as word processor, spread sheets and graphics have been found to alleviate some students' misconceptions of some mathematical and scientific ideas (Ghazali and Ismail (1997), and Fuglestead (1997)). Subject specific programs, i.e., programs designed specifically for a subject area such as derive, MATHEMATICA, MATLAB, etc. have also been found to be significant in rapid delivery of knowledge than traditional ways involving "chalk and board" or "paper and pencil" in mathematics instruction (Kaino (1994, 1998)). The *Computer Algebra Systems* (CAS), was also found by Murphy (1999) to easily generate graphical representations of problems, manipulate algebraic as well as compute numerical representations. TI-Interactive is another software to teach and learn mathematics. Some of its characteristics such as the automatic updating of documents and the existence of sliders allowing to change the values of parameters, makes TI-Interactive an excellent tool to illustrate different situations that eventually occur in a given problem (Gossez (2004)). Mathcad is a program that allows individuals to make numerical and symbolic calculations and graph expressions in 2D or 3D. It can be used to communicate technical ideas in a unified, easy-to-use and visual format (Grode (2004)). Combined in a seamless integration with Smartsketch program, Mathcad also performs computer-aided precision drawing and diagramming. On its own, Smartsketch works both as an interactive dynamic geometry application and as a computer aided design application.

Dynamic Geometry is a software described to enhance the drawing tools and methods for learning geometry. Cuoco and Goldenberg (2004) show how dynamic geometry environments can help students develop habits of mind such as "reasoning by continuity" that were useful in calculus and analysis. Given a dynamic diagram, a way can be found to reconstruct the same diagram with the condition of preserving its behavior. Moving the diagram can be a new way of exploring the problem and of finding solutions. Mathematical properties involved in dynamic geometry diagrams emerge in a different way than in paper and pencil drawings. Using Dynamic Geometry, Soury-Lavergne (2004) established the links between the technical use of the tool and the new way mathematical properties appeared to the learner.

Dynamic Geometry software is now widely used in teaching and learning of geometry; and the software's facility for easy visualization and powerful manipulation of diagrams has extended its application to other areas of the mathematics curriculum, for example algebra, calculus, and trigonometry (Jackiw (2004)). One of the dynamic geometry software widely used is The Geometer's Sketchpad. With *Sketchpad*, students can construct an object and then explore its mathematical properties by dragging the object with the mouse. All mathematical relationships are preserved, allowing students to examine an entire set of similar cases in a matter of seconds, leading them to generalizations. *Sketchpad* encourages a process of discovery in which students first visualize and analyze a problem, and then make conjectures before attempting a proof (Key Curriculum Press (2004)). Some recent studies in the United States and Singapore have focused on new applications of *Sketchpad* to investigate continuous geometric representations and shapes at primary school level (Jackiw (2004)). At primary school level, the focus is usually more on discrete mathematics such as number patterns and elementary number theory, fractions and early algebraic reasoning.

Logo software consists of a programming language derived from LISP, the language of artificial intelligence. *Logo*'s philosophy and basics of the language remain the same since its beginnings in 1960s. *Logo* involves writing of structured programs, breaking down a problem into smaller tasks, learning about branching and conditionals and manipulating numbers, words and lists. Though 'old', *Logo* programs are still said to provide a rich and accessible framework for doing mathematics (Maswera and Tsvigu (2002)). The above cited literature indicates that a number of computer programs that do not use programming language such as *logo* or *fotran* in instruction have been developed and will widely be used in future.

Fotran is another language widely used in mathematics teaching and learning. Some current debates argue that with 'mushrooming' of such programs, the tendency would be to disregard programming language in learning, which is also important in the enhancement of process skills in mathematics learning. Whether the latter view holds much weight or not, in the process of imparting knowledge, remains to be evaluated by education researchers.

Activities

Rotational symmetric figure is a topic in BGCSE Mathematics syllabus and the specific objective is to recognize and describe rotational symmetry. The activity in the classroom is initialed with the assumption that mathematics teachers should be able to use computers to create different rotational figures. With that idea in mind we have designed activities for students' recognition and description.

Title

Construct a rotational symmetric figure by rotating the element about an angle several times.

Objectives

1. Understand that a rotational symmetric figure can be formed by rotating the element (branch) for n times, where n is the order.
2. Establish the relation between the order and the angle of rotation, i.e., order $\times$ angle = 360°.
3. Using computer to implement 1, and 2, in order to create rotational symmetric figure with
   - MSW Logo.
   - Geometer's Sketchpad.

Activity with MSW Logo

1. Use two procedures, one called *element* for creating the element, and the other called *rotate* for rotating the element for n times, each time the angle of rotation is $360^\circ/n$.
2. Execute rotate with the value of n.
3. Testify with various values of n.
4. Modify the procedure element to change the shape of element, enhance to obtain different rotational symmetric figures.

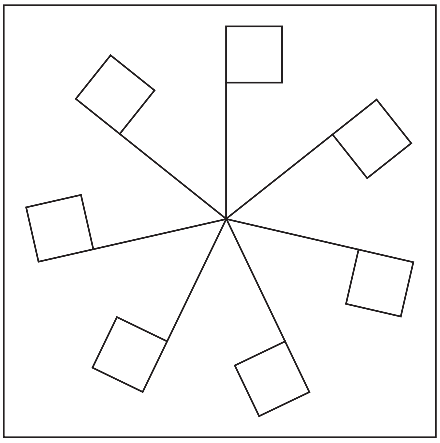
Procedures

```
to element
fd 70
repeat 4[fd 30 rt 90]
bk 70 end

to rotate :n
repeat :n [element rt 360/:n]
end
```

Instruction to the turtle and resulting graphic

```
cs rotate 7
```



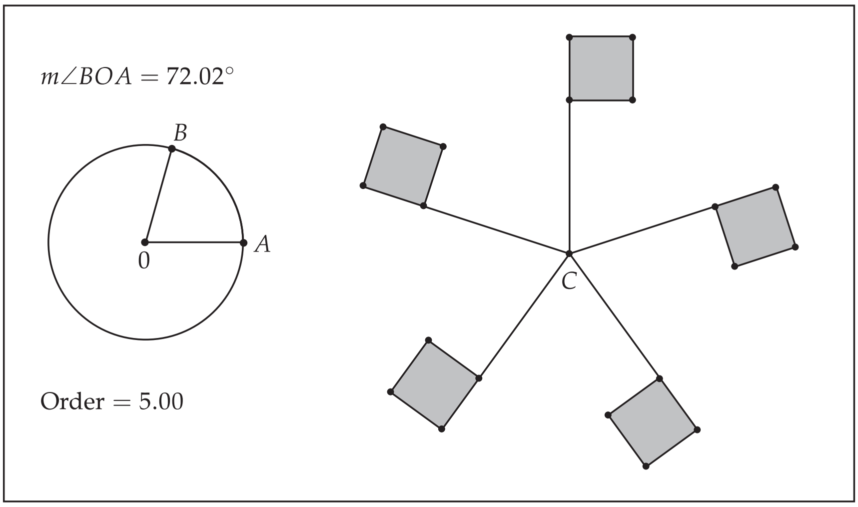
Activities with Geometer's Sketchpad

Activity 1. Rotate the element through any angle to find out when a rotational symmetric figure will be obtained

Step-by-step instructions of Activity 1:

1. Draw a circle at center O with radius OA.
2. Construct a point B on the circle.
3. Measure and mark the angle AOB.
4. Construct an elementary figure.
5. Create a point C, and mark it as the center.
6. Select the elementary figure and rotate it with the center C through the angle AOB.
7. Repeat 6.

8. Drag point B along the circumference of the circle.
9. Observe the figure formed by rotating and justify if it is a rotational symmetric figure.



Activity 2. Design a sketchpad to create rotational symmetric figure

Step-by-step instructions of Activity 2:

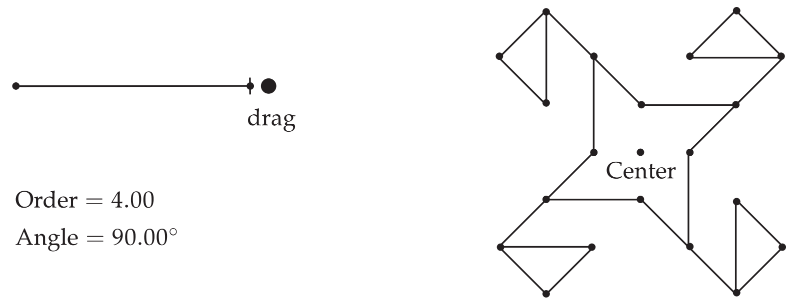
1. Open: *Program files/Sketchpad/Samples/Custom tools/Sliders*
2. Open a new sketchpad. Select *Custom Tool/Sliders/Integer Horizontal*.
3. Change the name of variable from a to order.
4. Calculate $\frac{360^\circ}{\text{Order}}$. Mark and label it as *Angle*.
5. Construct any polygon with 8 vertices and interior, ignoring the shape.
6. Construct a point. Mark and label it as *center*.
7. Select and rotate the polygon with the center through the angle.
8. Repeat rotating with *iterate* tool about 30 times.
9. Try to do the followings:

   - Drag the "drag circle" in the horizontal slider to change the order of rotational symmetry.
   - Drag any vertex of the polygon to change the shape.
   - Drag the "center" to change the position of the center.

Rotational symmetric figure

Try to do the followings:

1. Drag the "drag circle" in the horizontal slider to change the order of rotational symmetry
2. Drag any vertex of the polygon to change the shape
3. Drag the "center" to change the position of the center



Testification of hypothesis by Logo and Sketchpad

Given that n is a rational number, $n = \frac{p}{q}$, and $\theta = \frac{360°}{n}$, rotate any figure about any point as the center, through $\theta, 2\theta, \ldots, (p_1 - 1)\theta$, the entire figure obtained has rotational symmetry with order p_1, where $\frac{p_1}{q_1}$ is the simplest form of $\frac{p}{q}$.

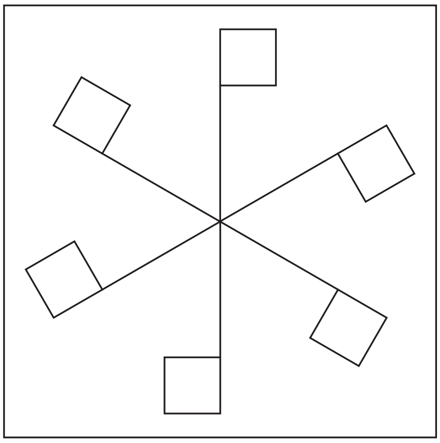
MSW Logo

Procedures

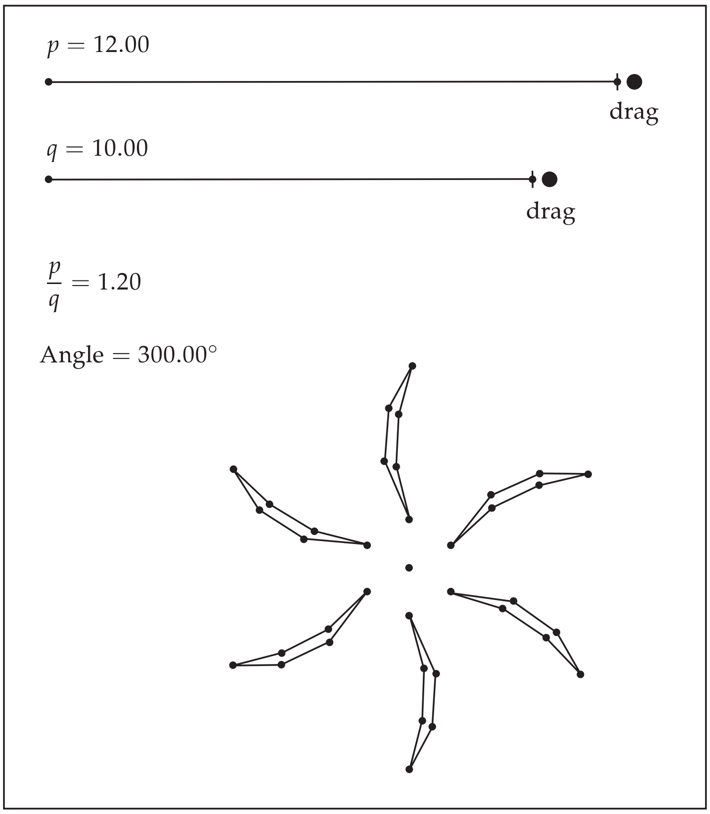
```
to element
fd 70
repeat 4 [fd 30 rt 90]
bk 70
end
to rotate :p :q
repeat :p [element rt 360/(:p/:q)]
end
```

Instruction to the turtle and resulting graphic

```
cs rotate 12 10 ht
```



Sketchpad



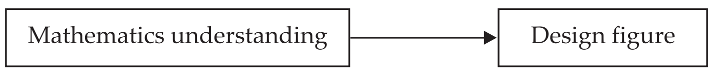
Summary

1. Using both MSW Logo and Sketchpad in instruction, enhances students' competencies of constructing rotational symmetric figures.
2. Both MSW Logo and Sketchpad allow students to implement their understanding of mathematics in constructing rotational symmetric figures.
3. Thinking skills required in activities differ from each other.
   - *Logo*: Logic analysis plays very important role in programming design. Mathematics understanding of the figure could be vital step of the program design.
   - *Sketchpad*: Visualizing thinking skills, such as using of UGI (*User's Graphic Interface*) are important. Sometimes it is possible to start construction without fully understanding of the task, see Activity 1. The first sketch could be a trail to find the relation between the order and the angle of rotational symmetric figure. However you can only create a good quality sketchpad when you have a well mathematics understanding of your task, see Activity 2.
4. Different approaches have been used in activities:
   - *Logo*: Create two procedures, element, and rotate. Type instructions to execute the procedure, as well as determining the variable. Obtain the resulting graphic. Try different value of variable. Change the element.
   - *Sketchpad*: Create a control angle and mark the angle. Create an elementary figure. Rotate the element at a center of the angle marked. Repeat the rotation. Observe the effects when change the control angle.
5. Interactivity and flexibility: The activities with both of MSW Logo and Geometer's Sketchpad could be designed in such ways that you can still intervene in the process of formation after the first results have been seen.
   - *Logo*: The order of rotational symmetric figure is a variable; therefore it is easy to create a rotational symmetric figure with any order by putting the value of the variable when running the program. The element is created with an isolated procedure element. Thus the change of the element can be done within the procedure element, without changing anything in the rest to produce various rotational

symmetric figures. Generally, in Logo, it is easy to testify the result for different discrete values of the variable.

- *Sketchpad*: It is easy to change the original element by dragging the vertices and all image elements will be automatically follow the change. An angle in Activity 1 and a slider in Activity 2 are controllers, by changing them different effects on the final figure are obtained. When you drag the center of rotation you may also wonder how many different figures could be produced. In sketchpad, it is easy to testify the results for a range of continuous values of the variable. Also it is possible to testify the results for different discrete values of the variable.

6. Prerequisites of doing the work with MSW Logo are mainly about programming, including: basic commands, such as, fd, bk, rt, repeat, cs, etc, basic syntax of Logo, use of variable, the structured programming techniques, etc. everything about construction is written in the program as commands. Prerequisites of using Sketchpad are: familiarity with working environment, tools bar, menu options, understanding of parent-children objects and ability to integrate all facilities to synthesize a dynamic sketchpad. Most of the time we just need to operate in the UGI.
7. Prospective development of the activity with MSW Logo: Pattern designing, see Activity 2. Sketchpad can be used to provide a huge number of raw materials where some new discoveries could be possible.
8. The involvement of mathematics in MSW Logo programming and creation of sketchpad could be dissimilar even though the topic of activities are the same. In this case, MSW Logo programming applies the mathematics about rotational symmetric figure construction. Concepts, relations and calculations are reflected in the procedures.



For the sketchpad, we first create an environment where the construction can be done. By interacting with the sketchpad we could describe mathematics facts, formulate mathematical relations between variables and apply explorations in creating more complex sketchpads.

Conclusion

The two softwares can be used to achieve the same objectives in a different manner. While Logo needed knowledge of creating the process using a traditional instructional strategy that was not rapid in delivery of knowledge, Sketchpad created a tool to design the process involving a variety of ways paving opportunity for more explorations in a student-centred strategy. Sketchpad's ability to involve both discrete and continuous problem activities was an advantage over Logo that utilized discrete values. In Logo, development of thinking skills required knowledge of programming language (which may not be easy to some learners), whereas Sketchpad does not require it.

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